Lazily Analysing the Result of Abstracting "Counting Processes"

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Goal

Verify safety of multi-threaded programs:

- involving an arbitrary number of threads (i.e. parameterized)
- synchronizing using "process counting variables", barriers, semaphores, etc.

Safety as in reachability of bad:

- individual configurations: assertion violation, mutual exclusion, etc
- global configurations : deadlock, number of threads at certain "states" is (e.g.,) smaller than the number of threads at some other "states", etc

Example

```
/* Only one main thread,
* no other threads    */

bool rd, op:=true;
int wt:=0,cn:=0;

main
{
  while(*)
  {
    atomic
    {
      spawn(thread);
      cn++;
    }
    op:=false;
  }
}
```

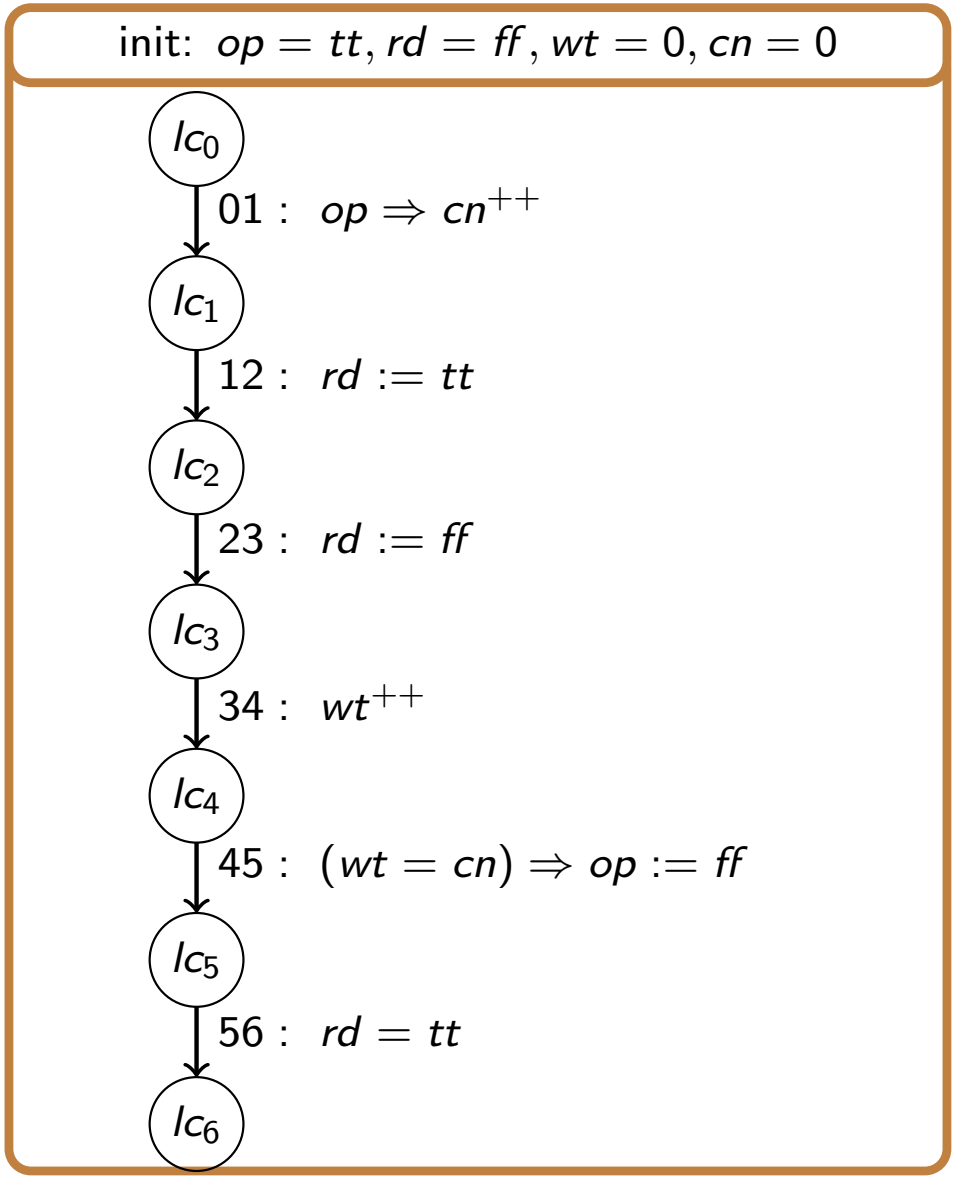
```
thread
{
  rd := true;

  //do read

  rd := false;
  wt++;
  assume(!op && (wt=cn));
  assert(!rd);
}
```

Example

- "Reformulation:" with identical threads.
- Initially, arbitrary many threads are at location lc_0
- Location lc_6 corresponds to the violation of the assertion



Goal and motivation

Predicate Abstraction

Strengthening

Monotonic abstraction

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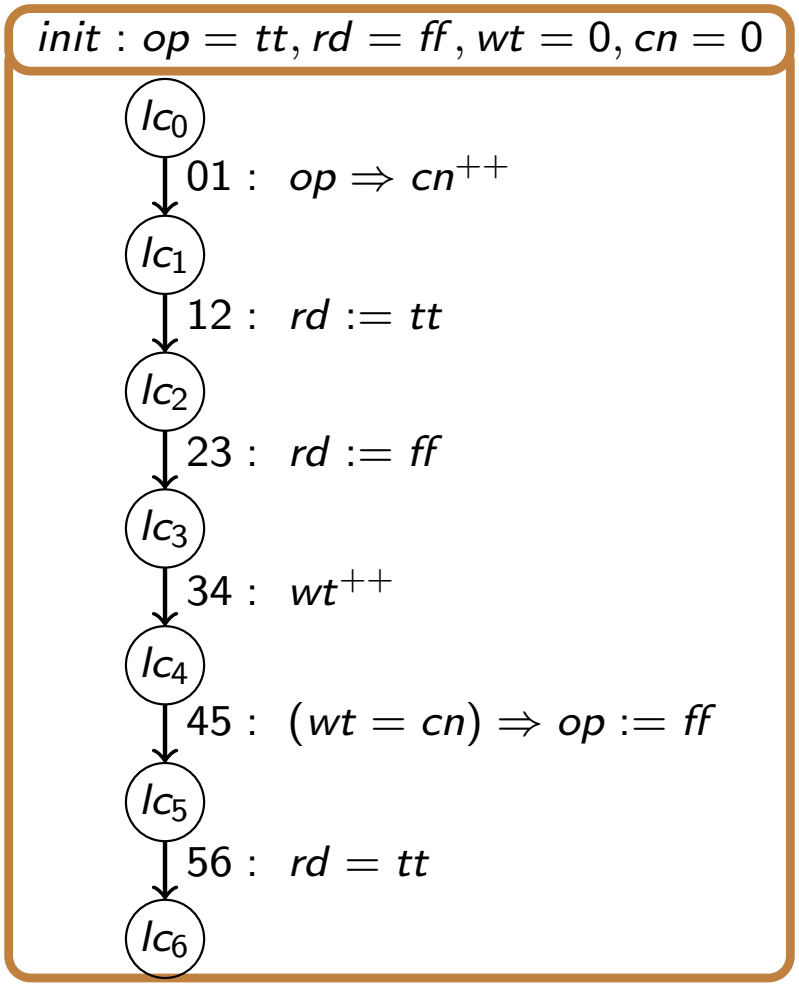
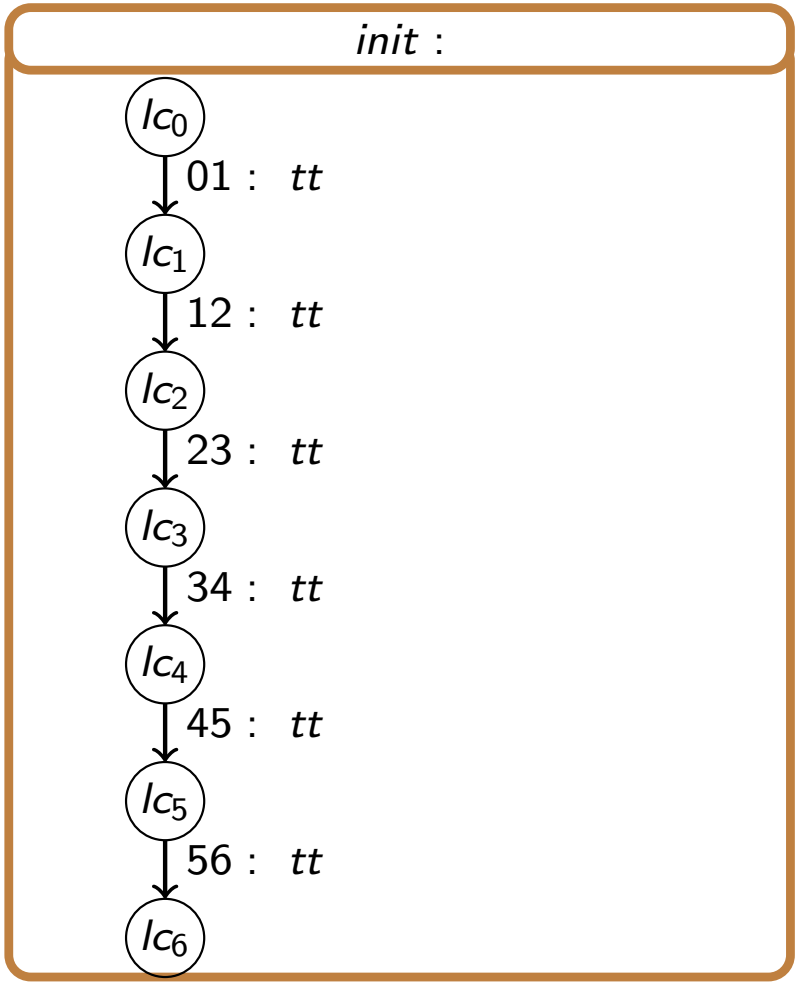
Predicate abstraction



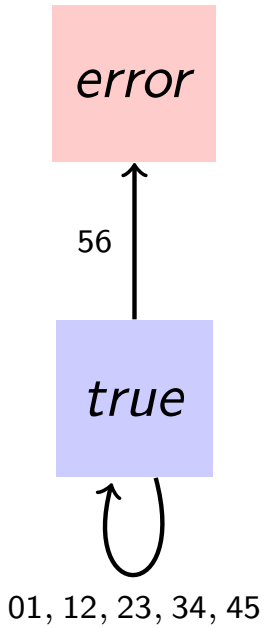


Counter Machine

Counter abstraction of the boolean program

- Control States
  - One state for each shared valuation of the Boolean program as well as error state
- Counters
  - One counter for each local valuation of the Boolean program
- Transitions
  - Translation of all possible transitions in the Boolean program
  - Local, global, transfer

Precisely captures the behavior of the boolean program


error
56
true
01, 12, 23, 34, 45

Counter Machine

Configuration

- State
- Value for each counter

Ordering on configurations

- $c_2 \geq c_1$ iff
  - Same state, larger counters in c_2

Closing upwards

- Intuitively amounts to adding more threads to a configuration

Counter Machine

A counter machine induces a well structured transition system, if

- Monotonic
  - The predecessor of every upward closed set is an upward closed set
- Well-quasi-ordering
  - Every increasing sequence of upward closed sets will eventually stabilize

Reachability Analysis

Check whether a control state is reachable from an initial configuration

Well structured systems:

- Classical backward reachability analysis

Reachability Analysis

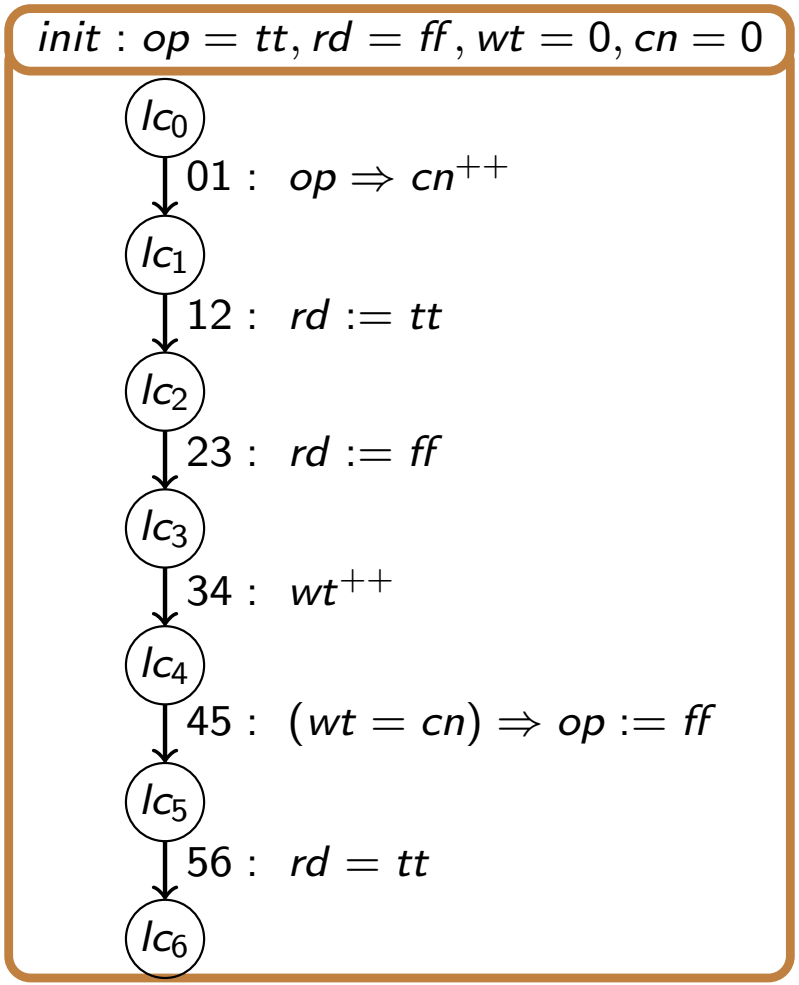
Check whether a control state is reachable from an initial configuration

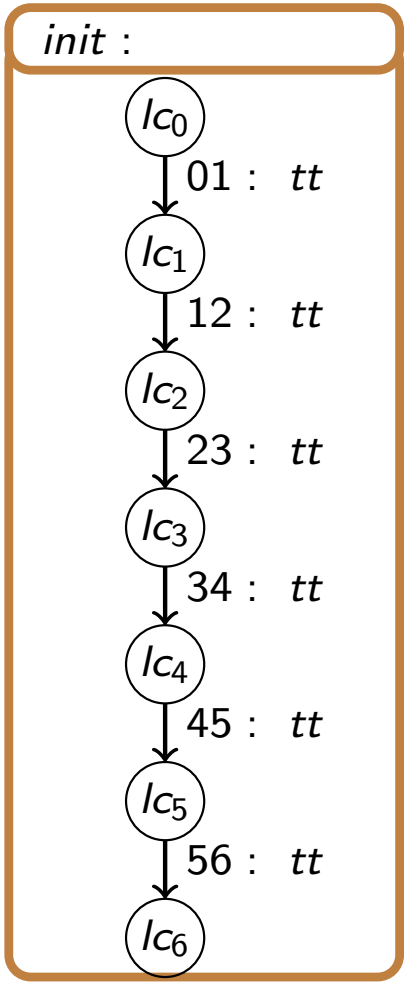
Well structured systems:

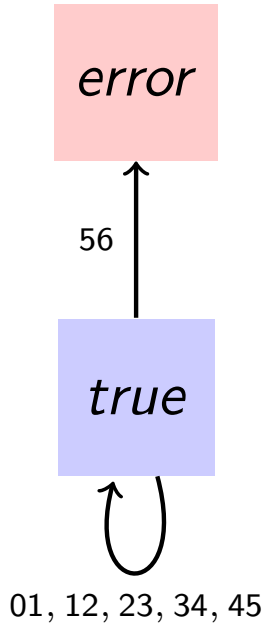
- Classical backward reachability analysis

  - Monotonicity: exactness
  - Well-quasi-ordering: termination

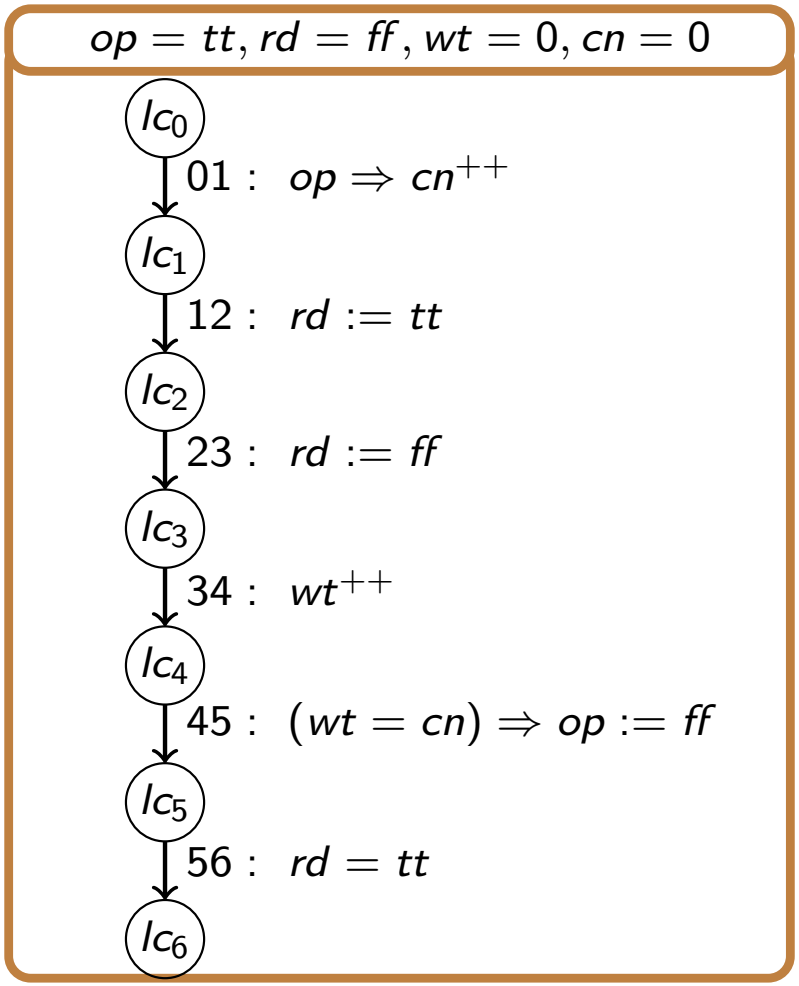
Predicate abstraction:

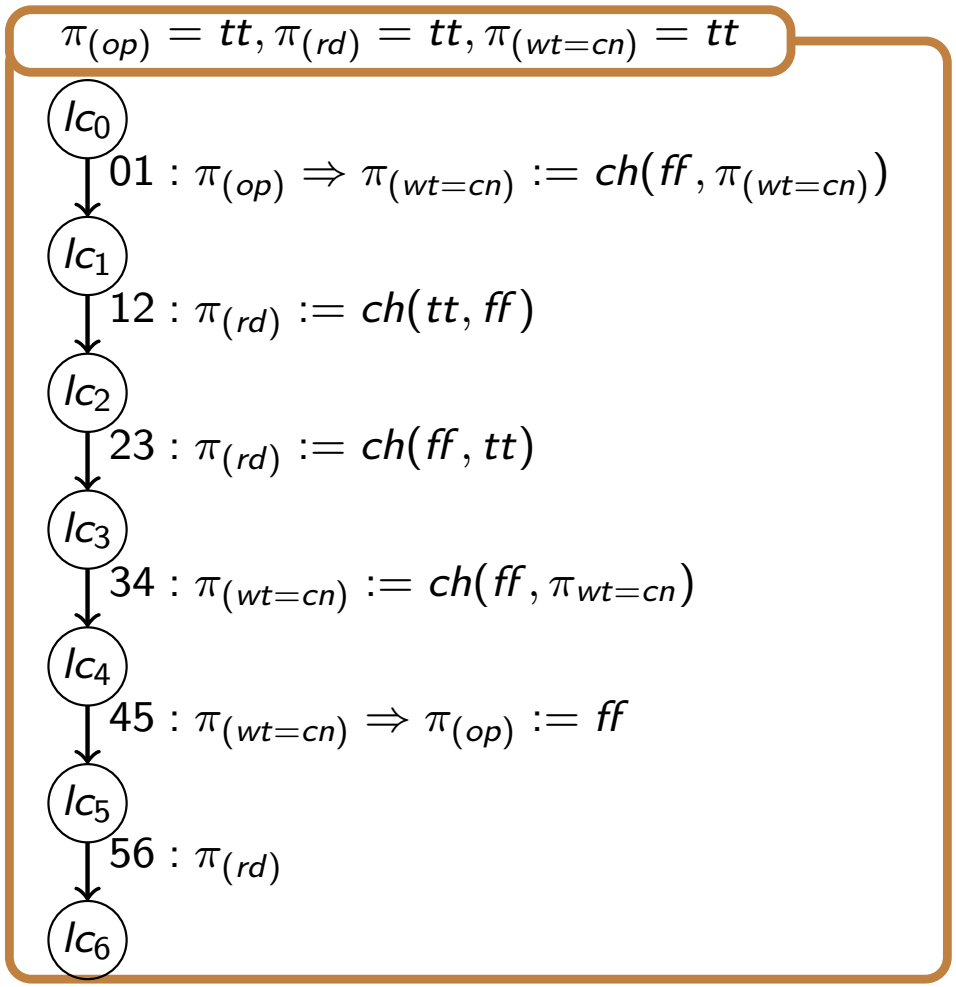


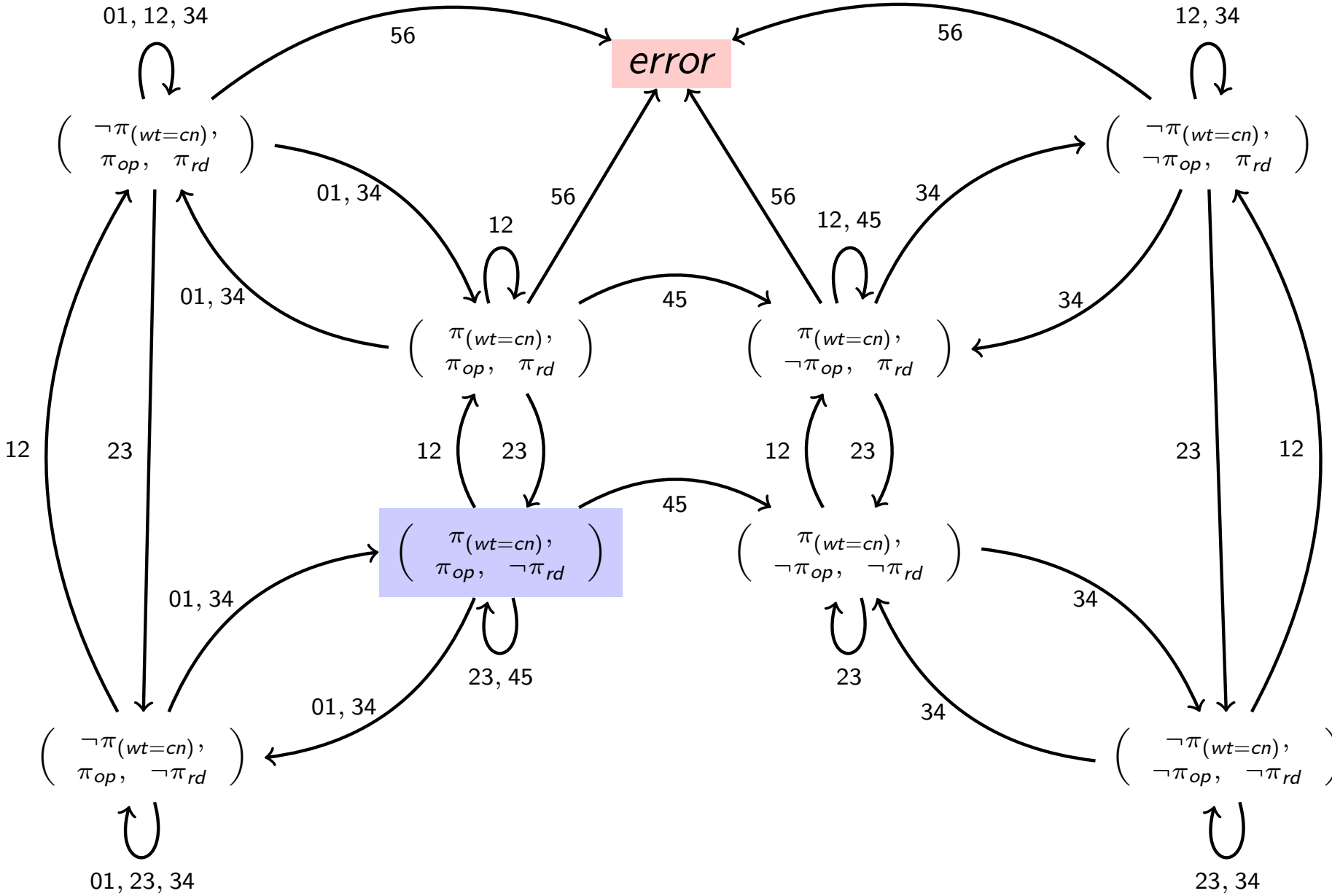




Predicate abstraction: (cont.)



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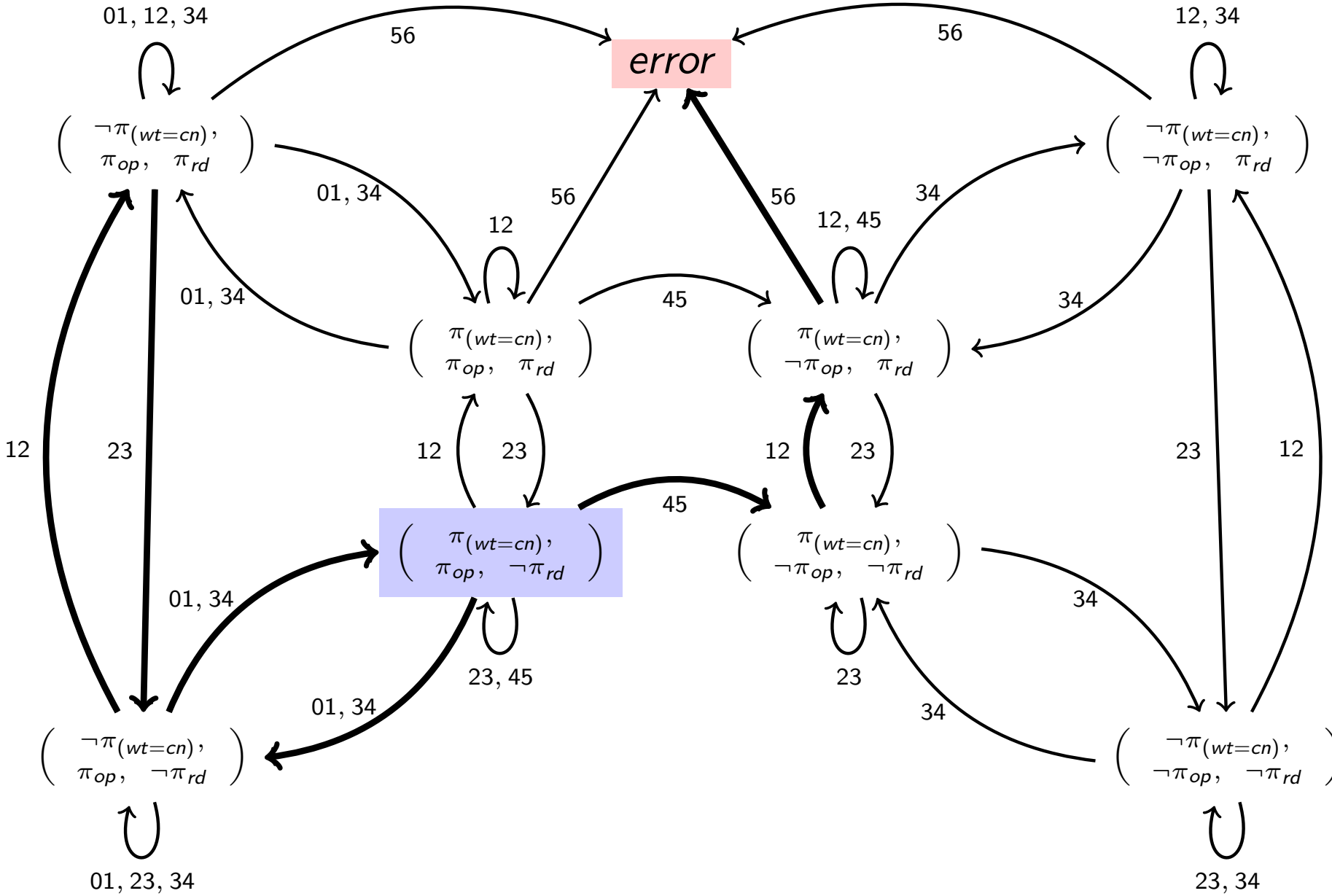
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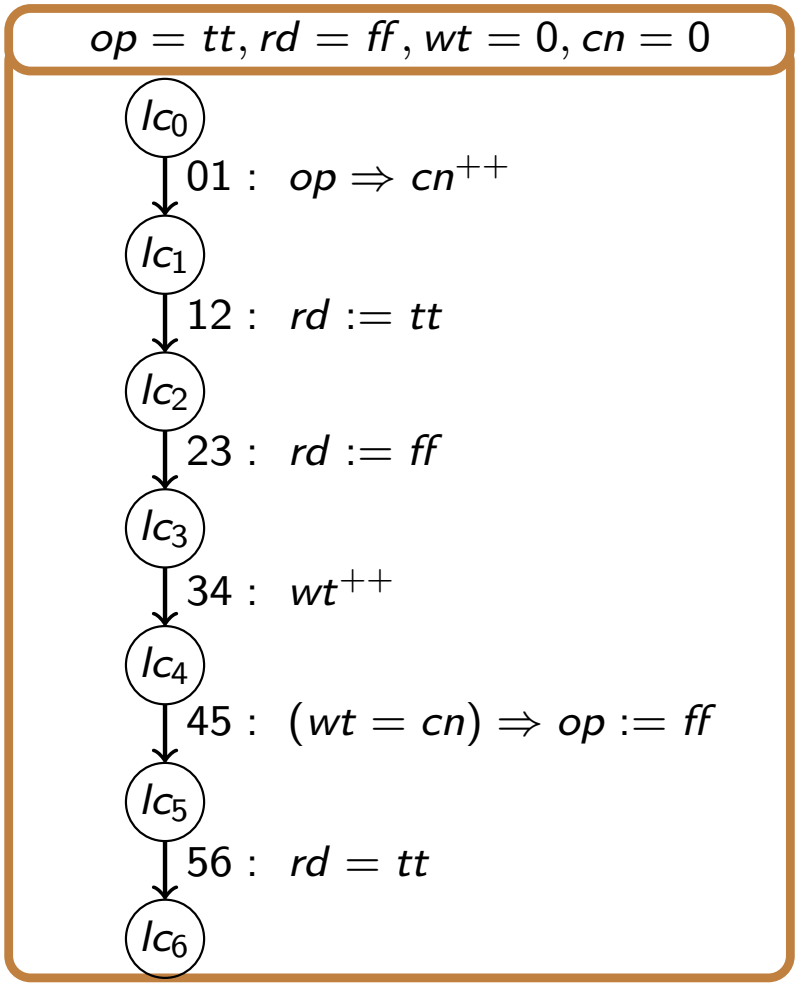
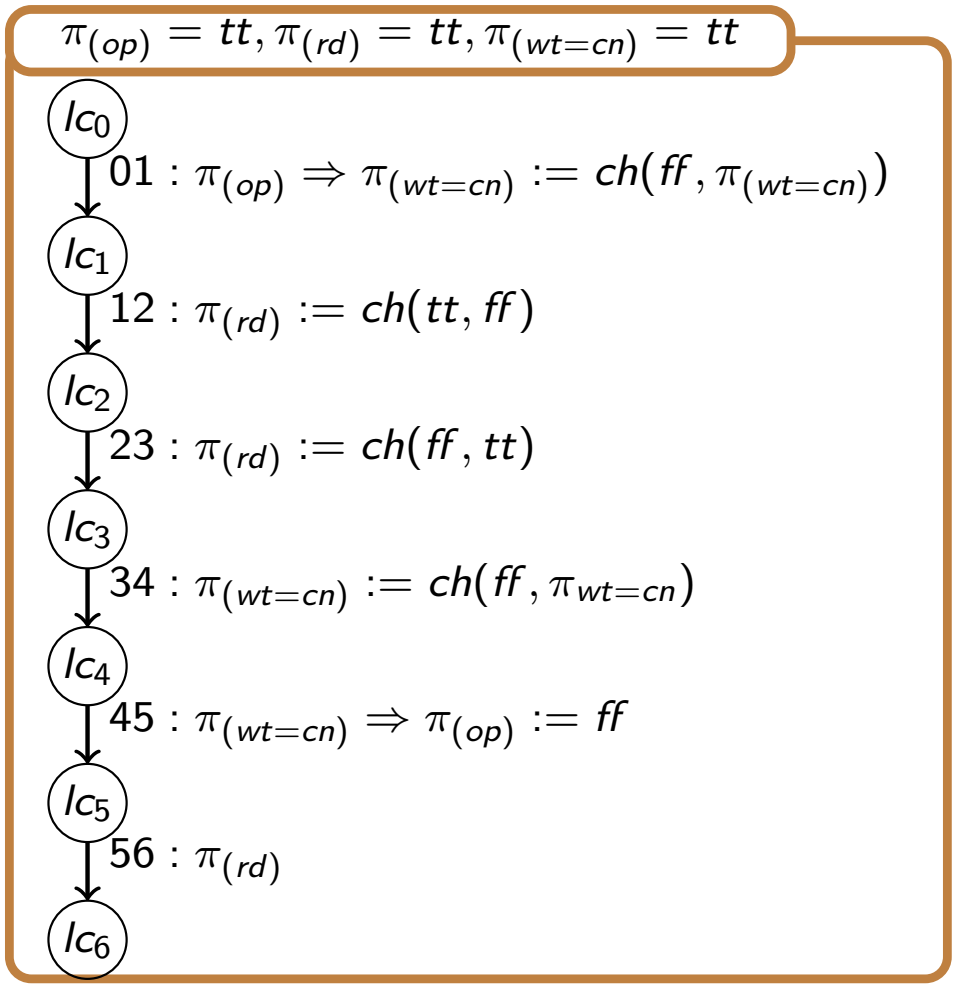
Predicate abstraction: (cont.)

The relation between the number of processes at locations lc_1, lc_2, lc_3 and the predicate $(wt = cn)$ is crucial (barrier).





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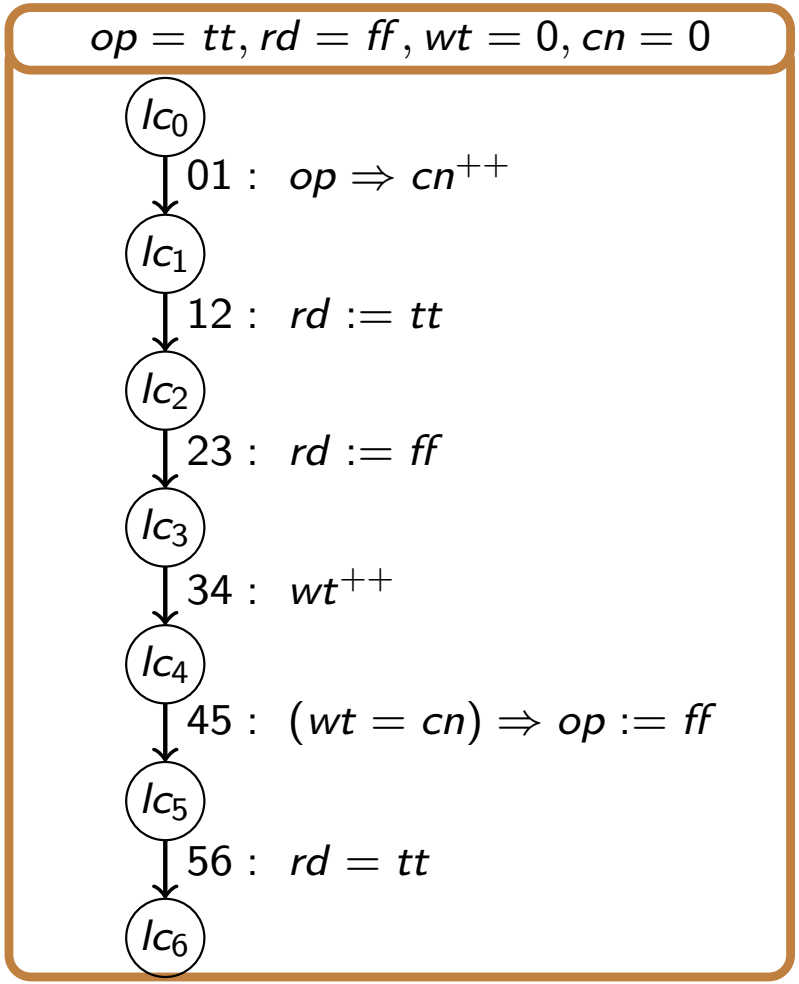
Counting Predicates

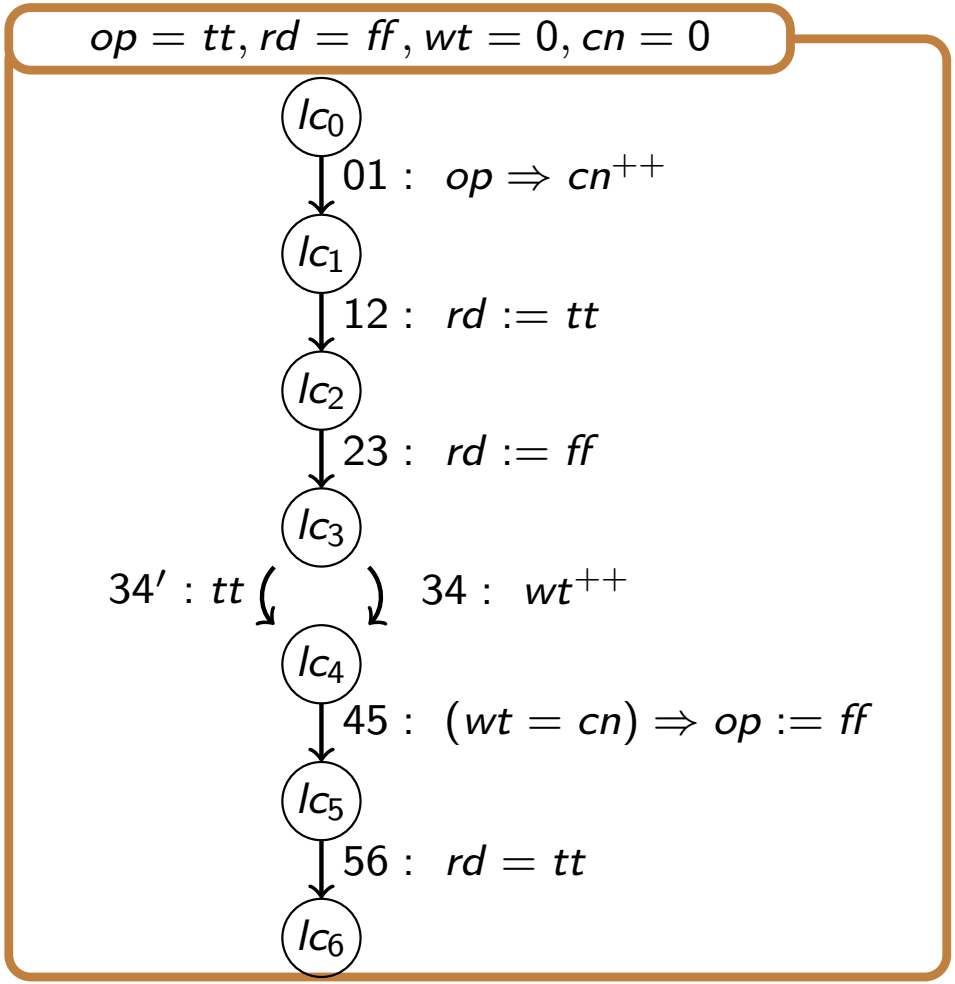
- Threads have states defined by their location and the predicates they satisfy on the program variables
- A counting predicate is a predicate that can involve:
  - shared program variables,
  - "ghost" counting variables $\mathcal{N}_{(\pi)}$ representing the number of threads satisfying some predicate π on the program variables
- We use them to define:
  - Properties to be checked, whether individual or global
  - Invariants that can be leveraged on during the analysis

Counting predicates for local properties:



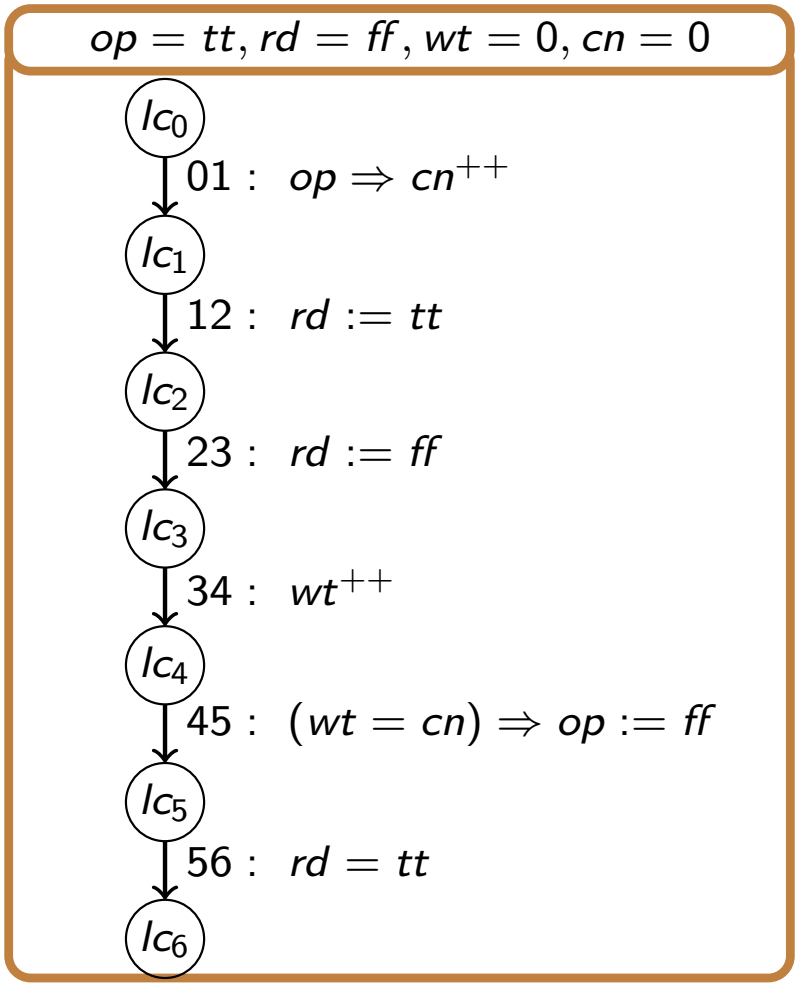
$\mathcal{N}_{(@lc_5 \wedge rd)} \geq 1$ reachable?

Counting predicates for global properties:



$\left(\Sigma_{i \neq 4} \mathcal{N}_{(@lc_i)} + \mathcal{N}_{(@lc_4 \wedge op \wedge wt = cn)} = 0\right)$ reachable?

Strengthening with Counting Predicates as Invariants



An appropriate thread modular analysis gives:

Invariants

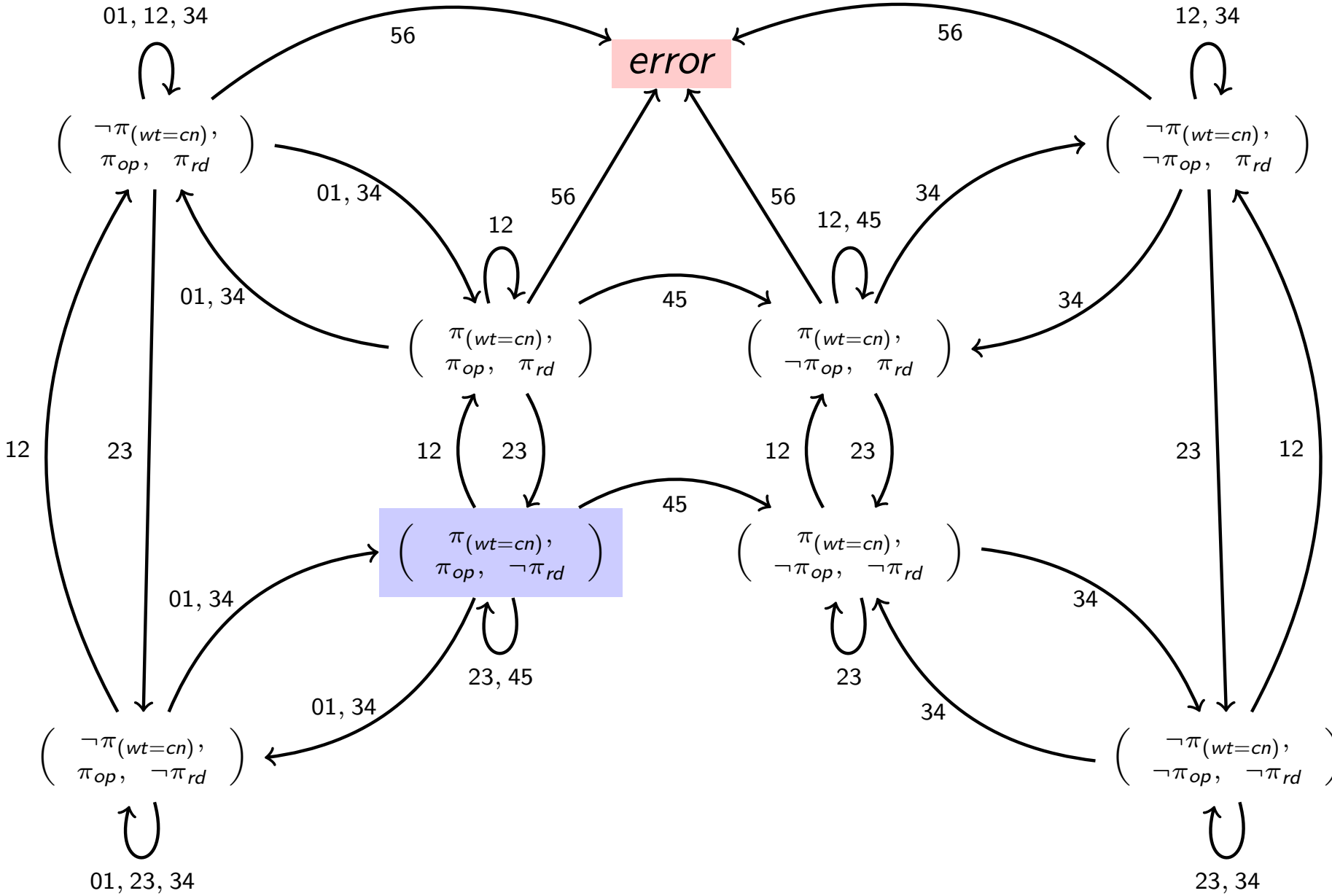
$cn = \sum_{i \geq 1}(\mathcal{N}_{(@lc_i)})$

$wt = \sum_{i \geq 3}(\mathcal{N}_{(@lc_i)})$

So each time ($wait = count$):

$\mathcal{N}_{(@lc_1)} + \mathcal{N}_{(@lc_2)} + \mathcal{N}_{(@lc_3)} = 0$

...source of non-monotonicity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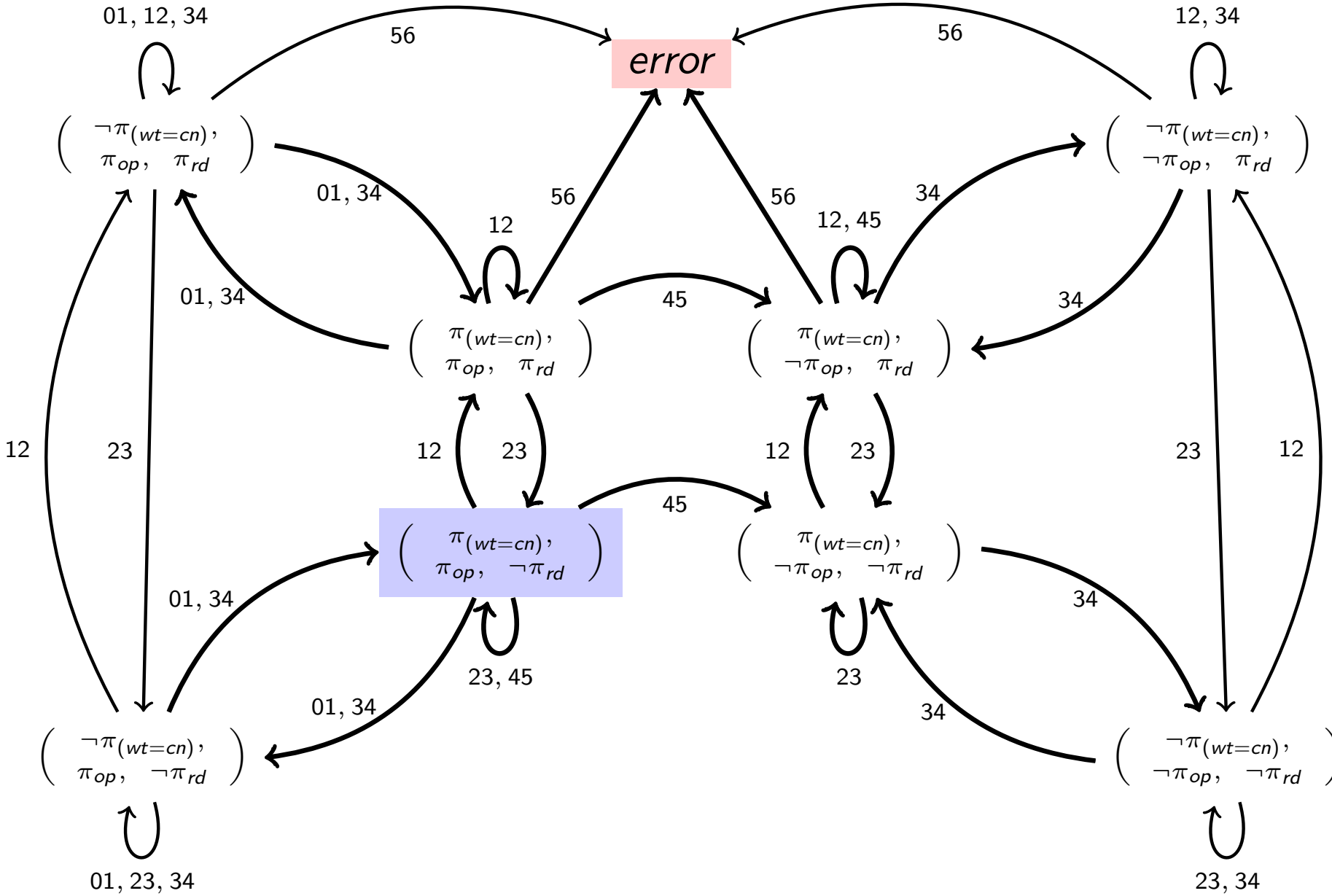
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Global Configurations

- Checking reachability of global configurations amounts to adding tests in the counter machine,
- For deadlocks we test that the number of enabled threads is zero,
- This makes the system non-monotonic

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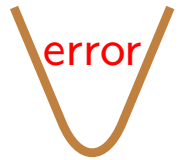
Backward Reachability Analysis



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lc_1

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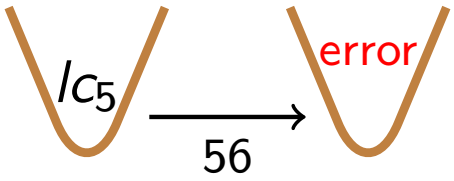

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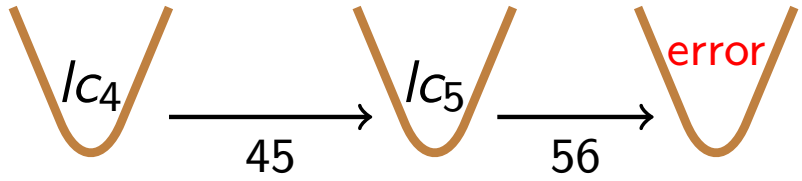

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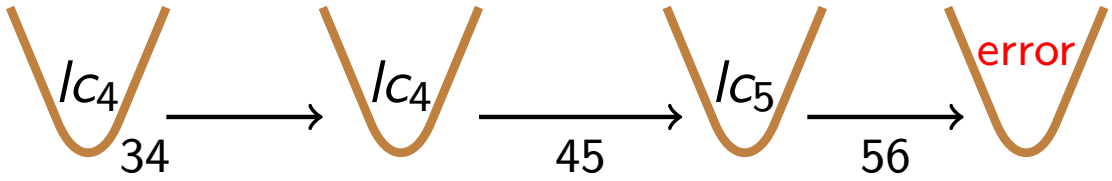

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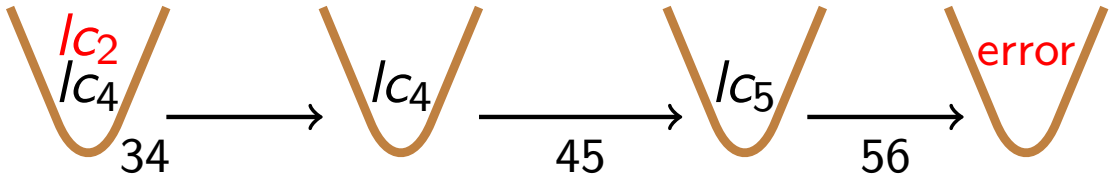

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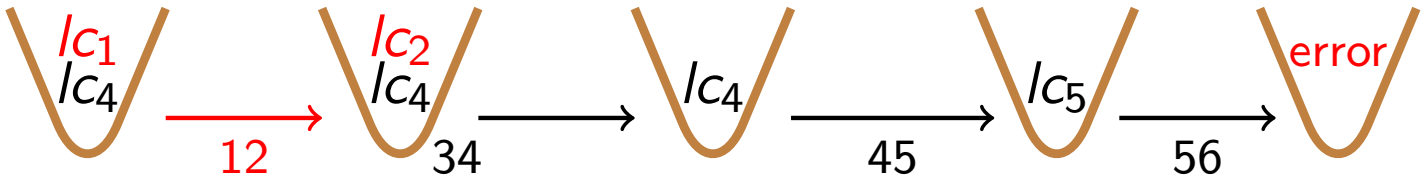

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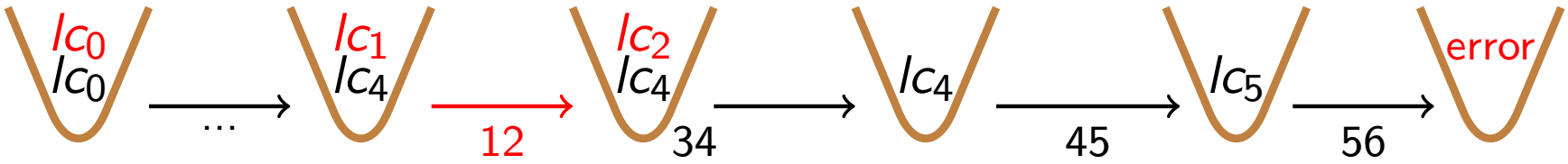

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PACMAN: PredicAted Constrained Monotonic AbstractioN