Temporal Deductive Verification of Parametrized Systems

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IMDEA Software Institute

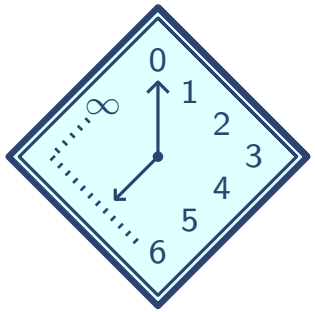
Joint work with Alejandro Sanchez

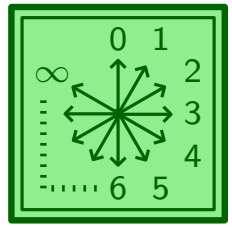
2nd Int'l Workshop on Parametrized Verification Madrid, 4 Sep, 2015

The Problem

The Problem

Temporal Properties





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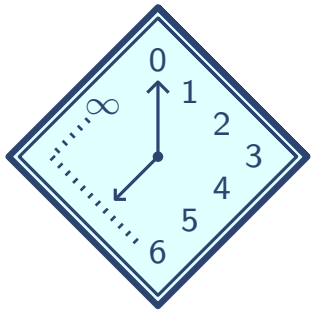
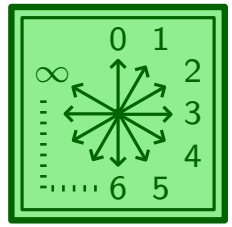
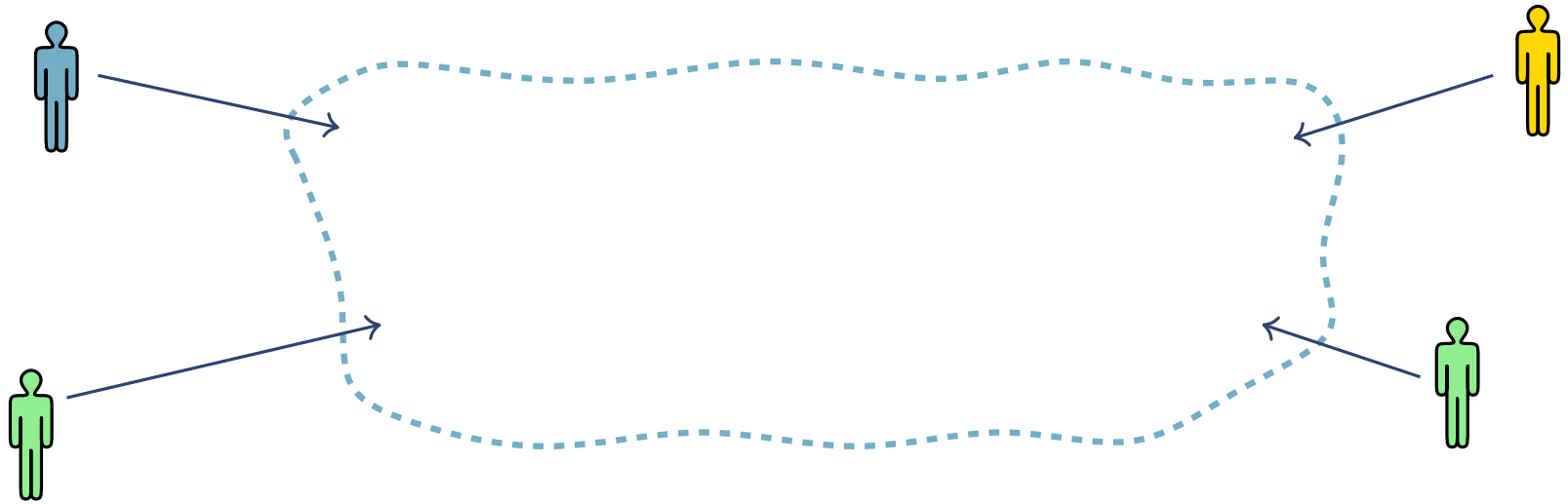




Concurrent Datatypes



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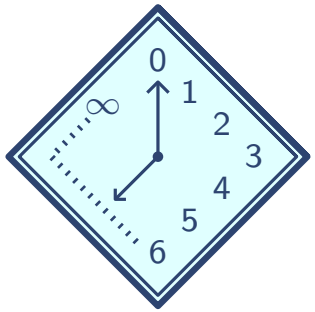

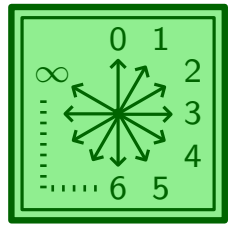

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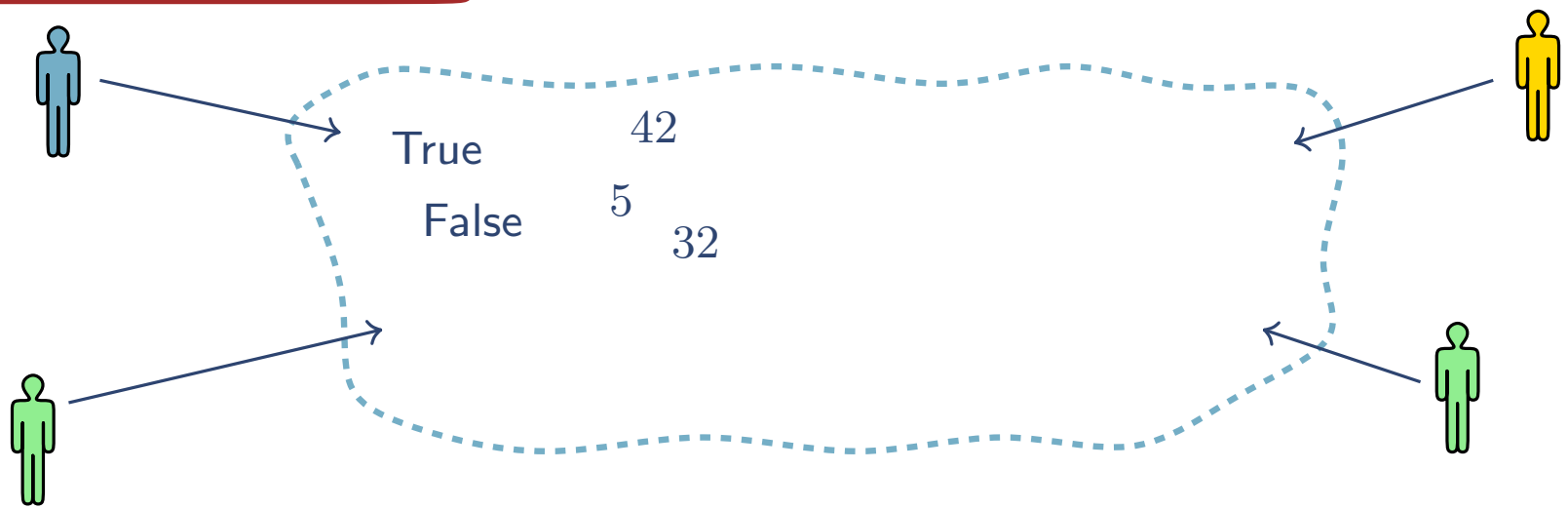

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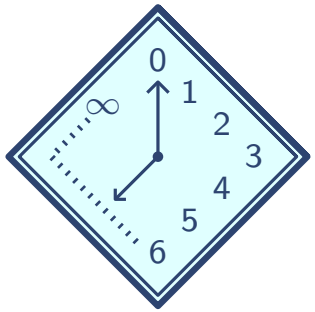

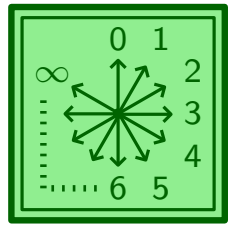

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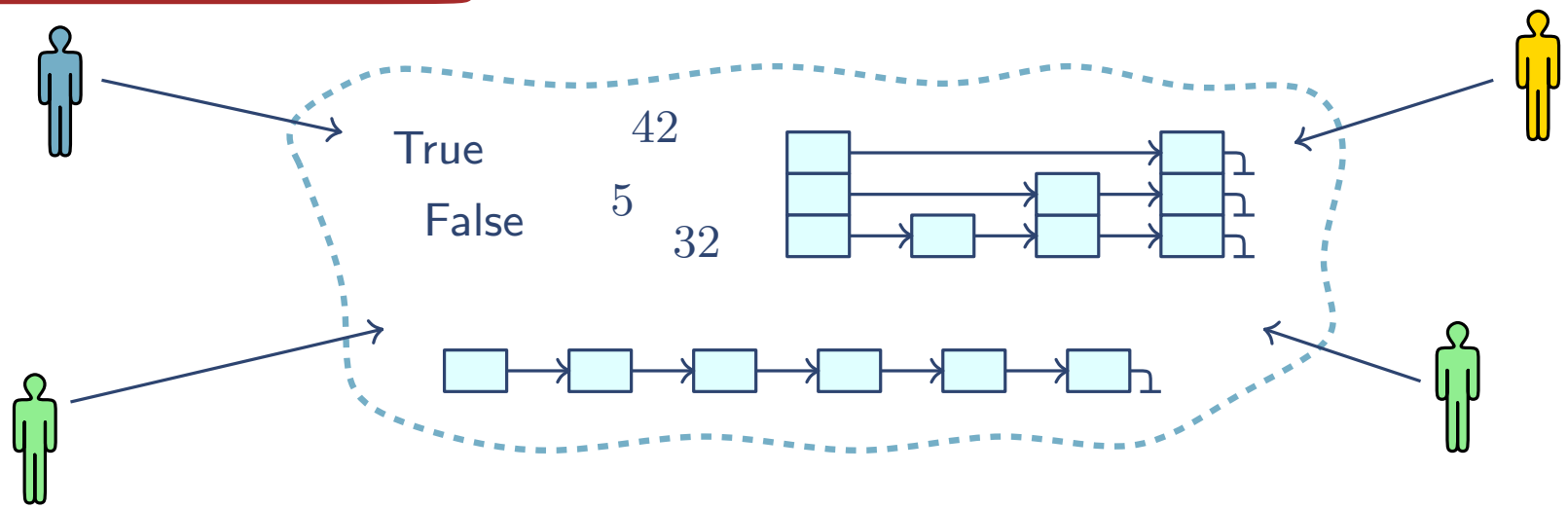

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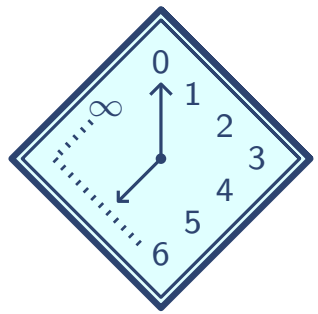

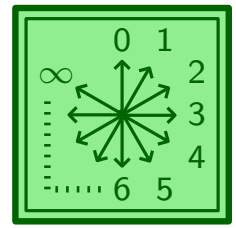

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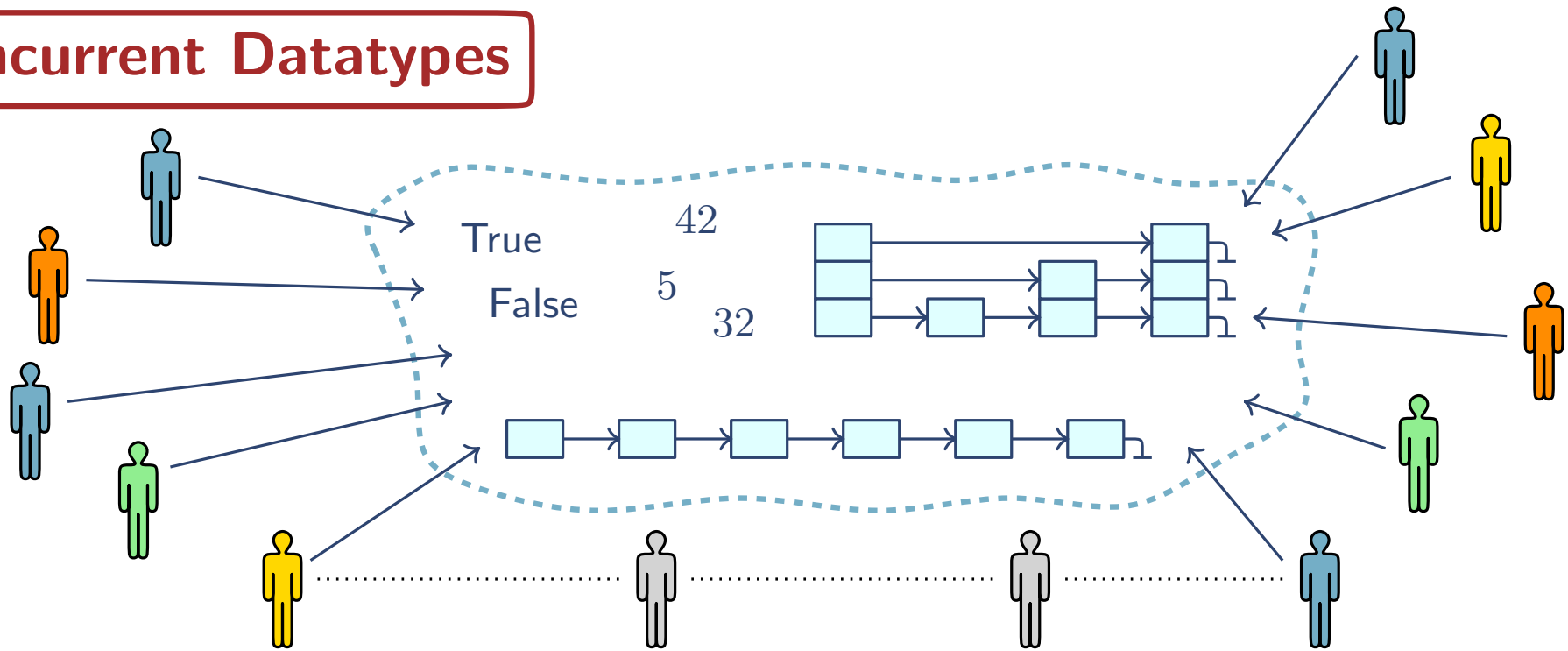
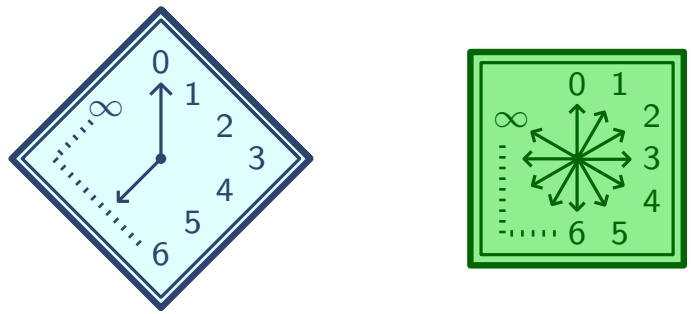


Parametrized Verification

The Problem

Temporal Properties



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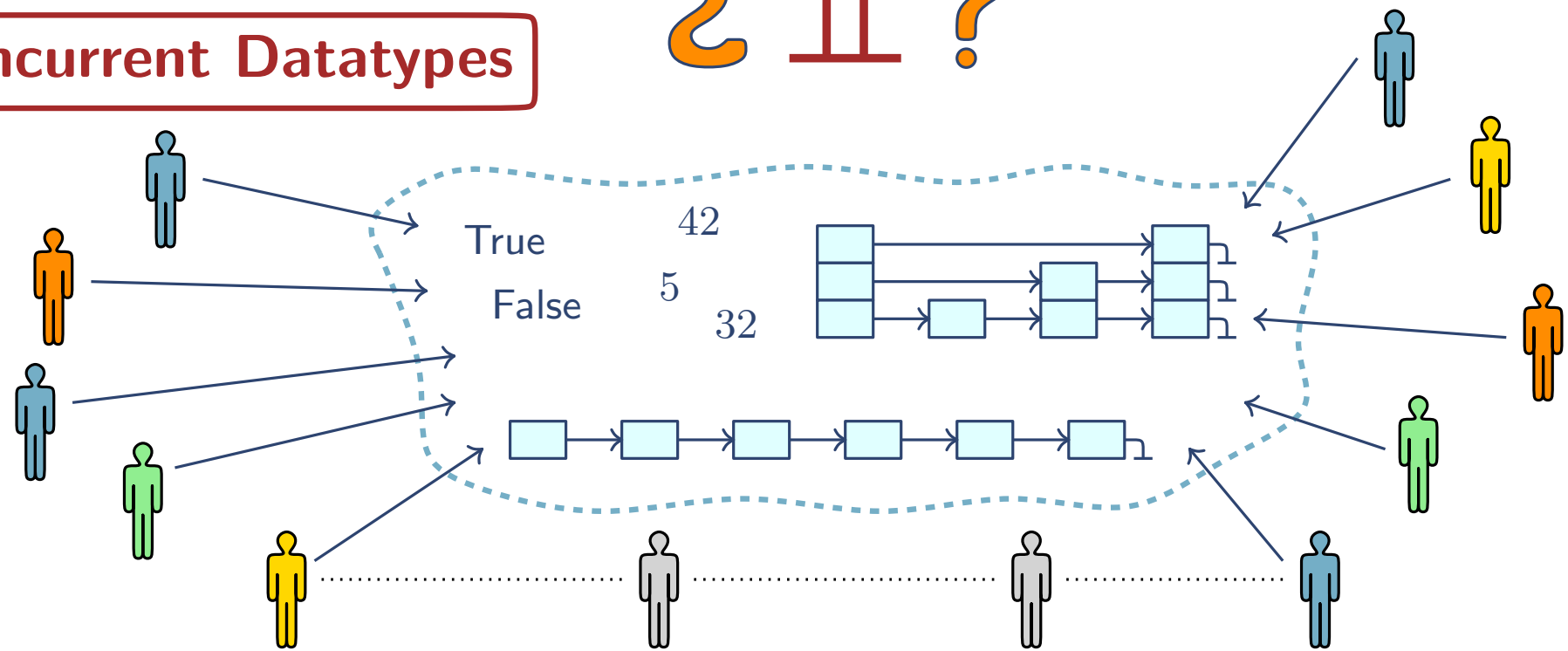
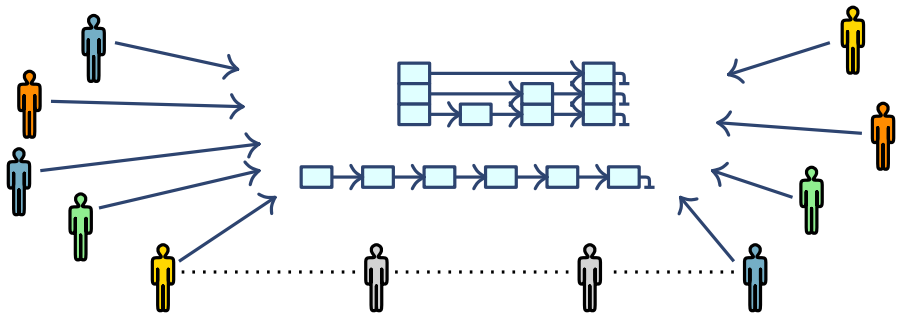


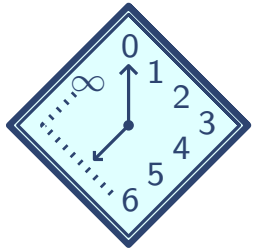
Parametrized Verification

Motivation

- Goal: **Formal Verification** (proving system correct)
- First target: concurrent data structures
- **Complex data types**: lists, trees, skiplists, hash-maps, etc
- **Unstructured fine-grained** synchronization methods
- **Automation** preferable but not mandatory
- Need to tackle **lock-based** and **lock-free** synchronization
- **Liveness** is as interesting as **safety** properties
- Parametrization to enable verification for **all system instances**

Our Approach





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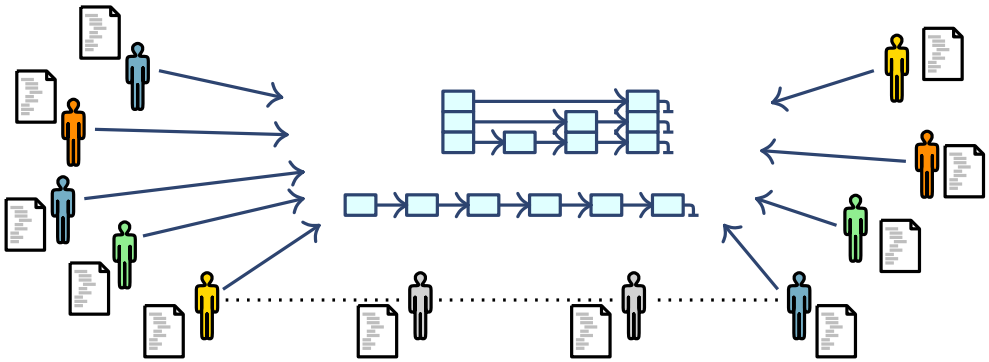

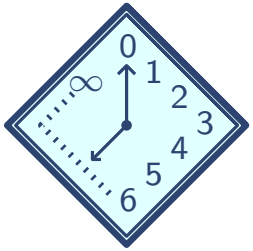

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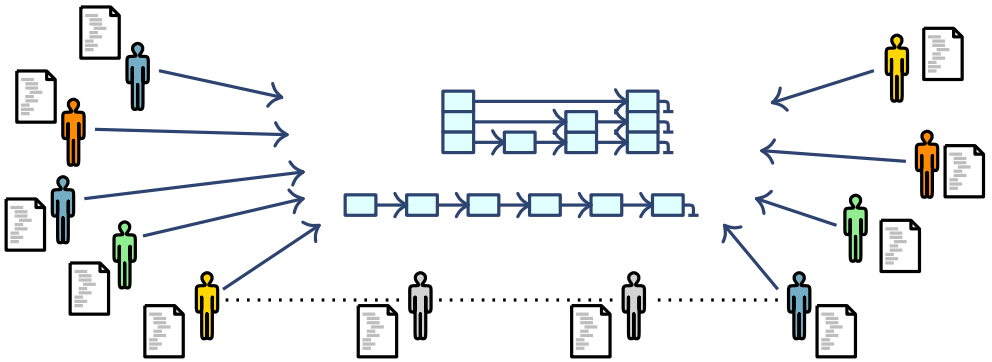
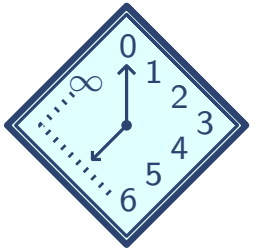


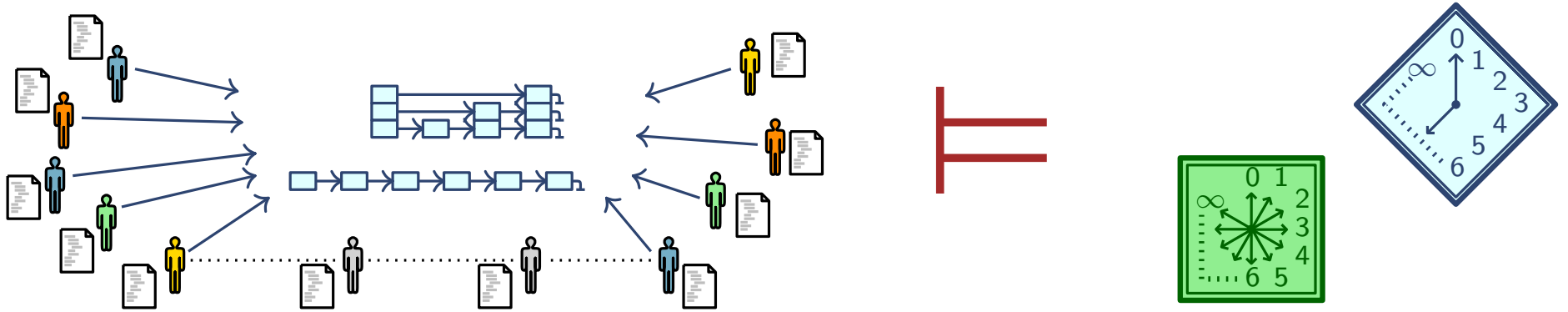


Safety

Liveness

Parametrized Deductive Methods

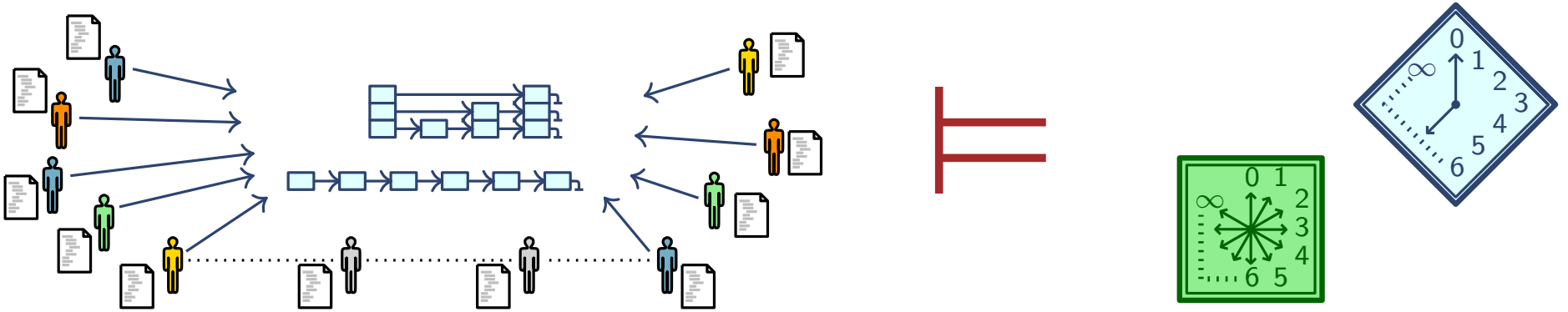
Our Approach



Parametrized Deductive Methods

Safety	Liveness
Parametrized Invariance	**Parametrized Verification Diagrams**

Our Approach



Parametrized Deductive Methods

Parametrized Invariant Generation	
Safety	Liveness
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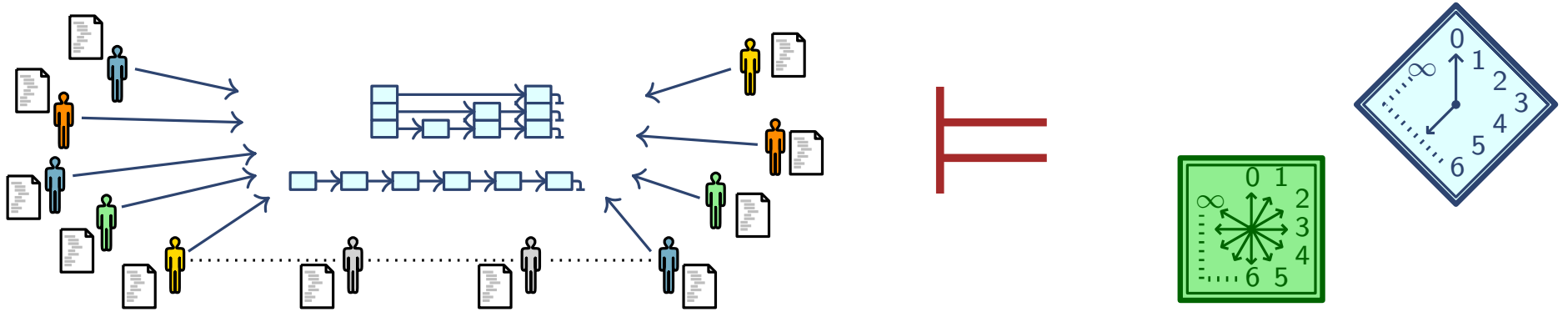

Parametrized Deductive Methods

Parametrized Invariant Generation

Safety	Liveness
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Finite collection of VC

Decision Procedures

Locations	Numeric	Lists	Queues	Stacks	Skiplists

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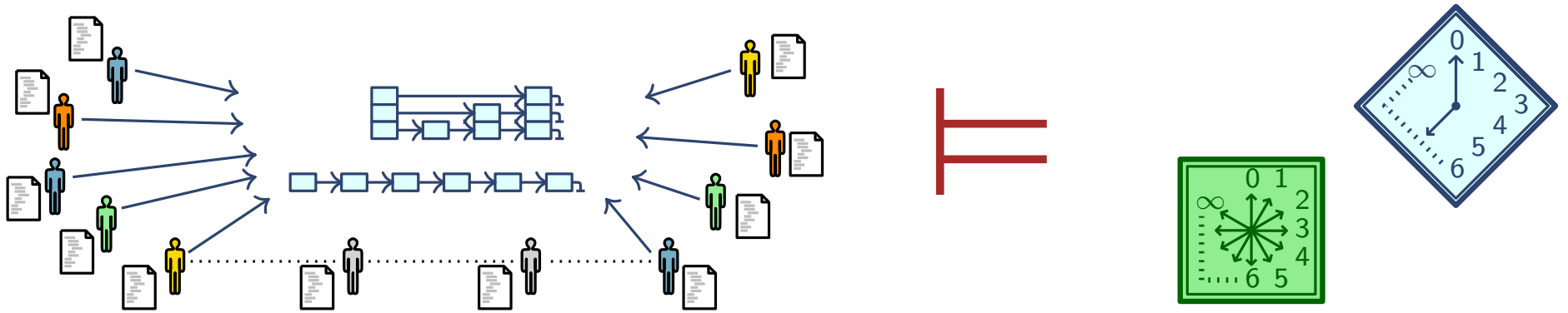

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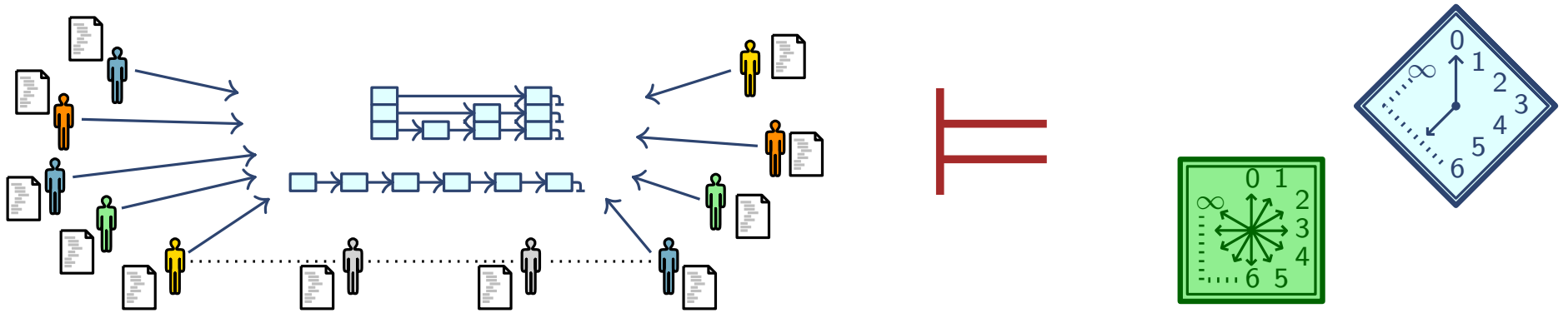

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SMT Solvers (Z3, Yices, CVC4,...)

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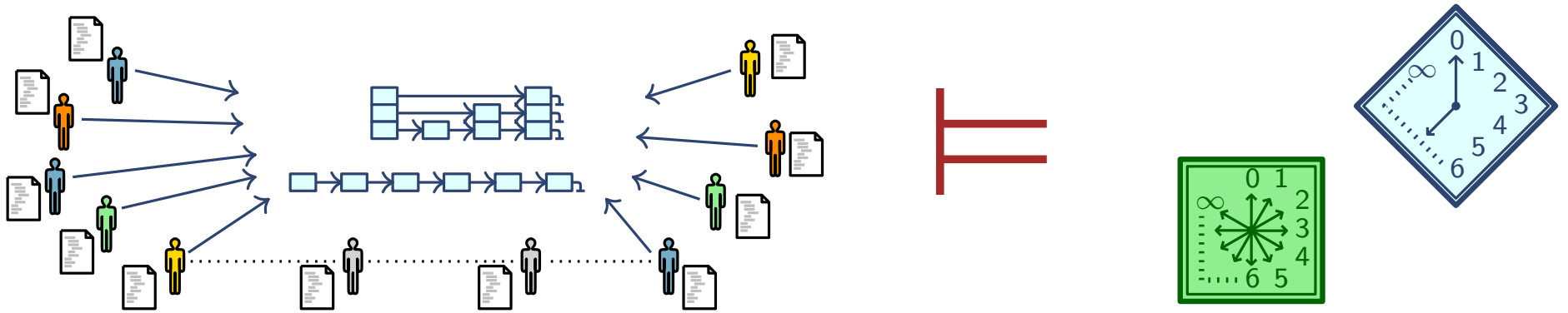

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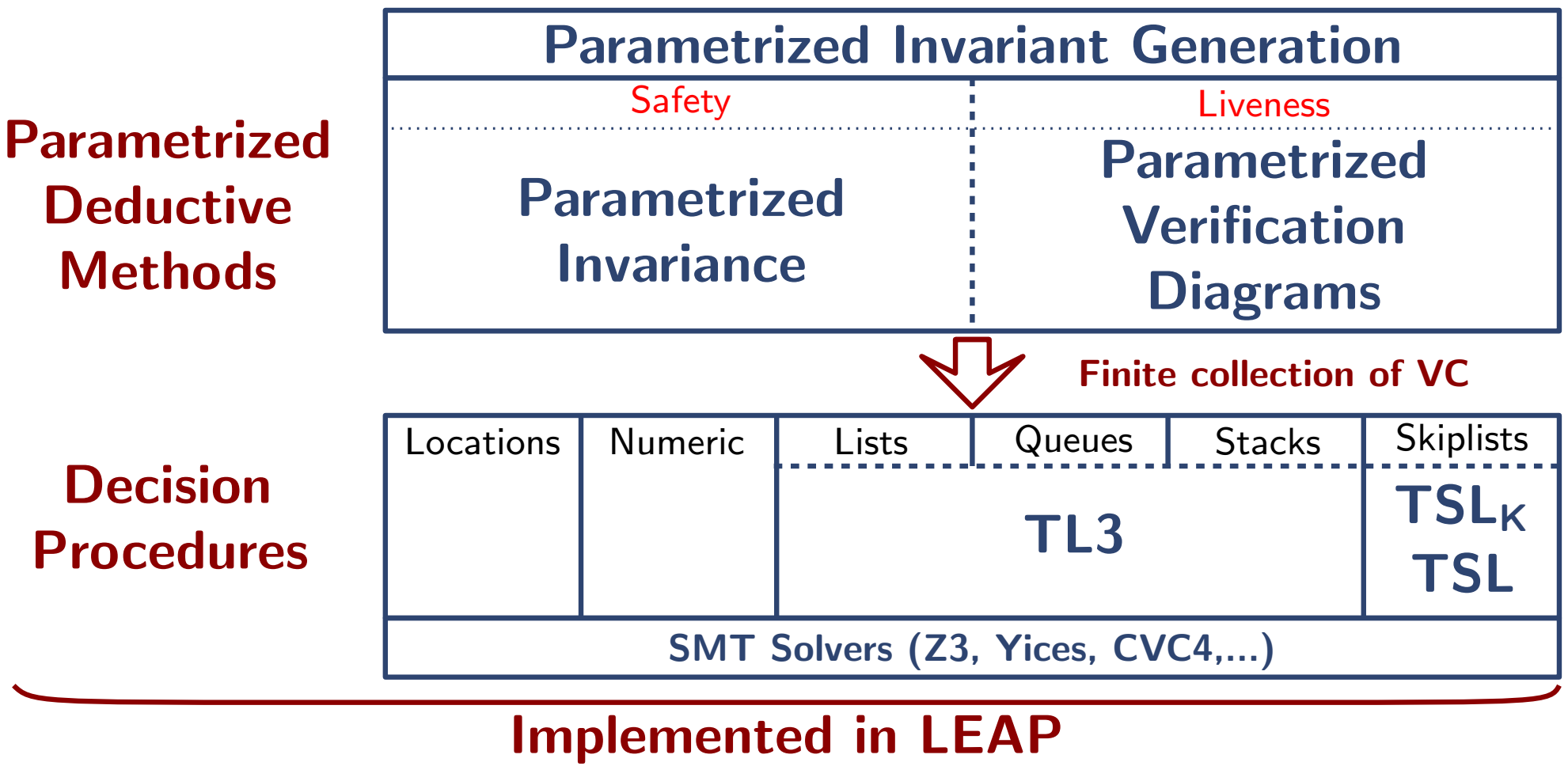
Implemented in LEAP

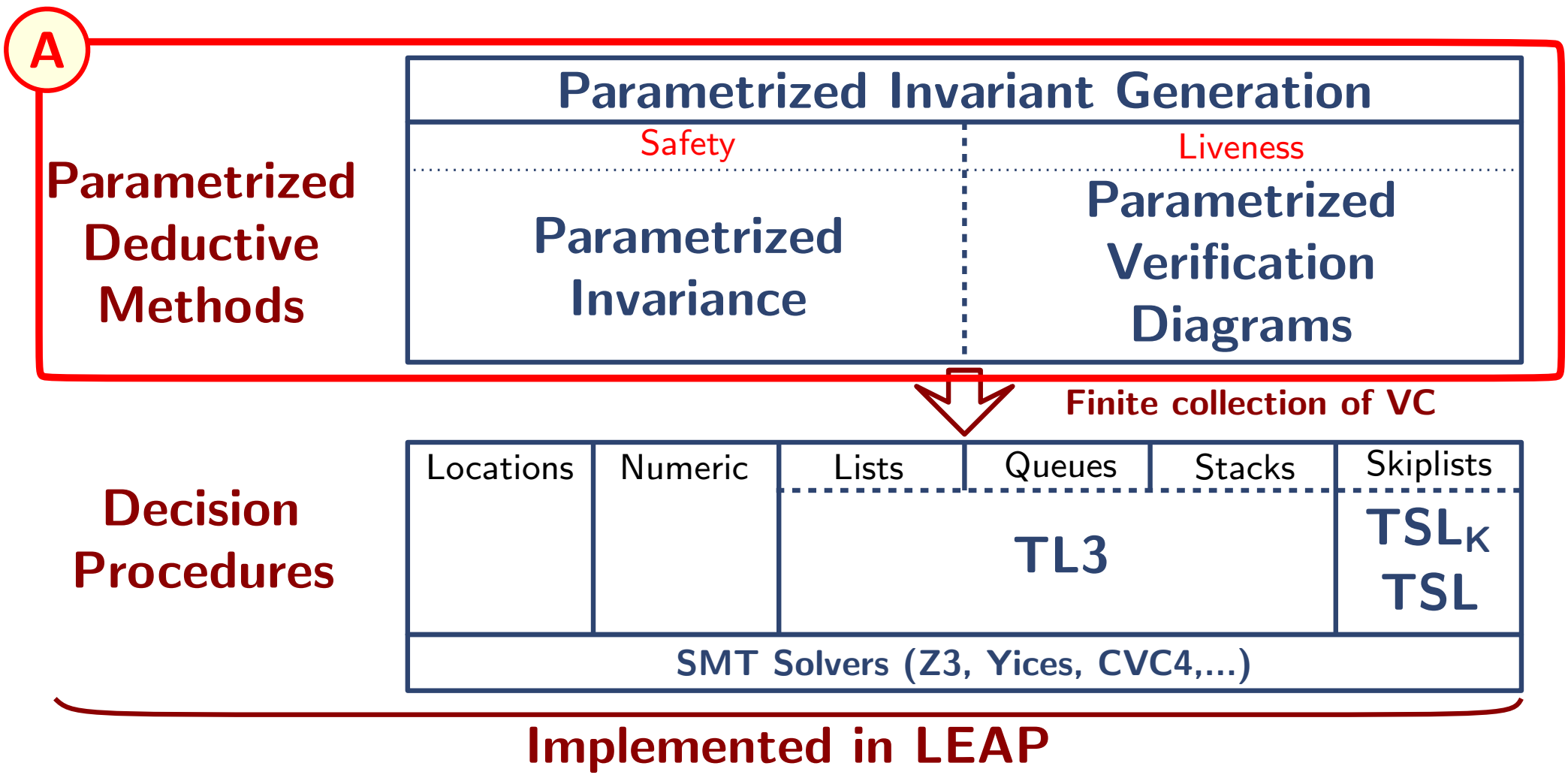
Our Contributions

A deductive verification framework for parametrized concurrent systems



Our Contributions

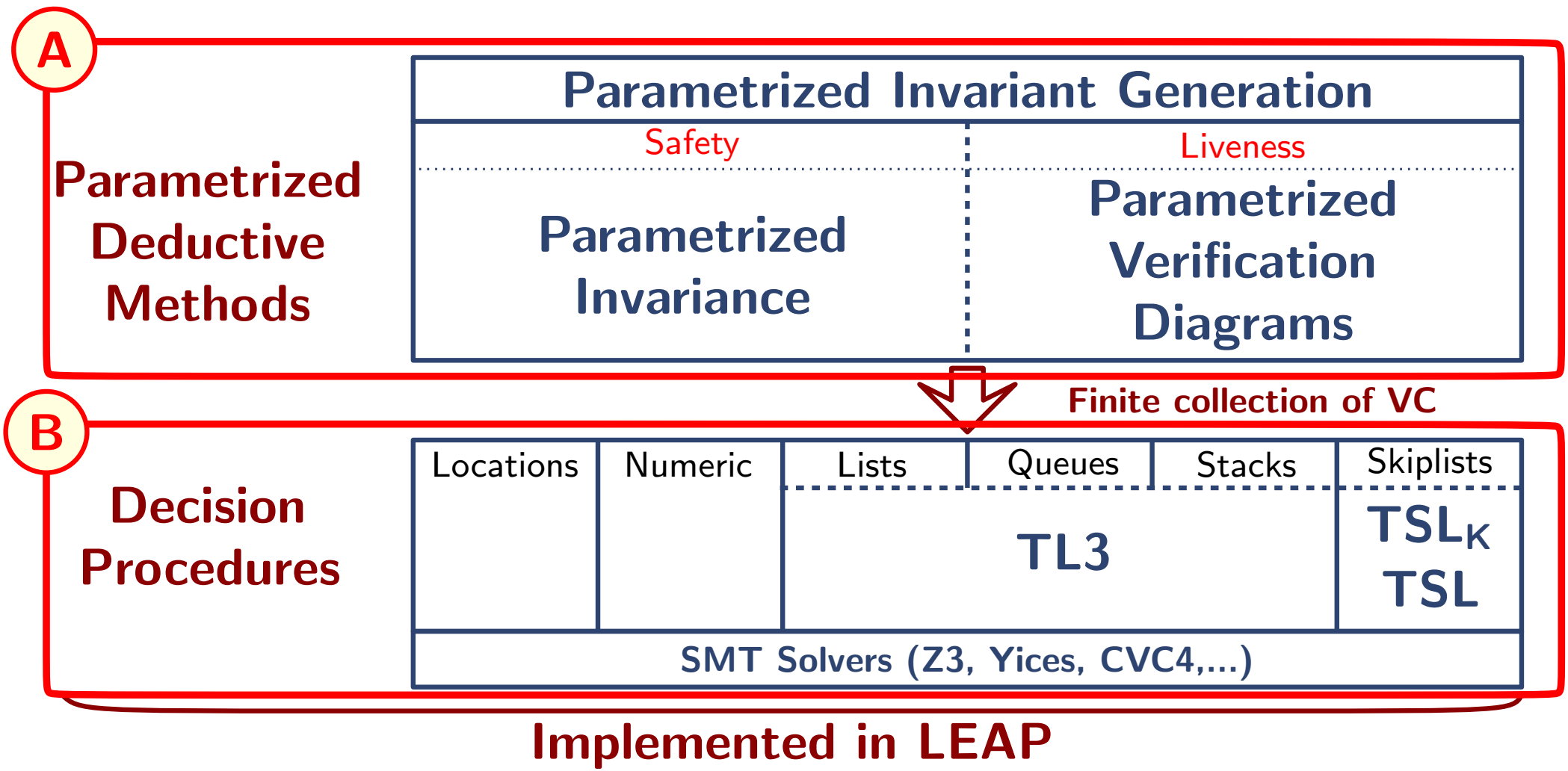
A Deductive Verification Techniques for Parametrized Systems



Our Contributions

A Deductive Verification Techniques for Parametrized Systems

B Decision Procedures for Complex Data Structures

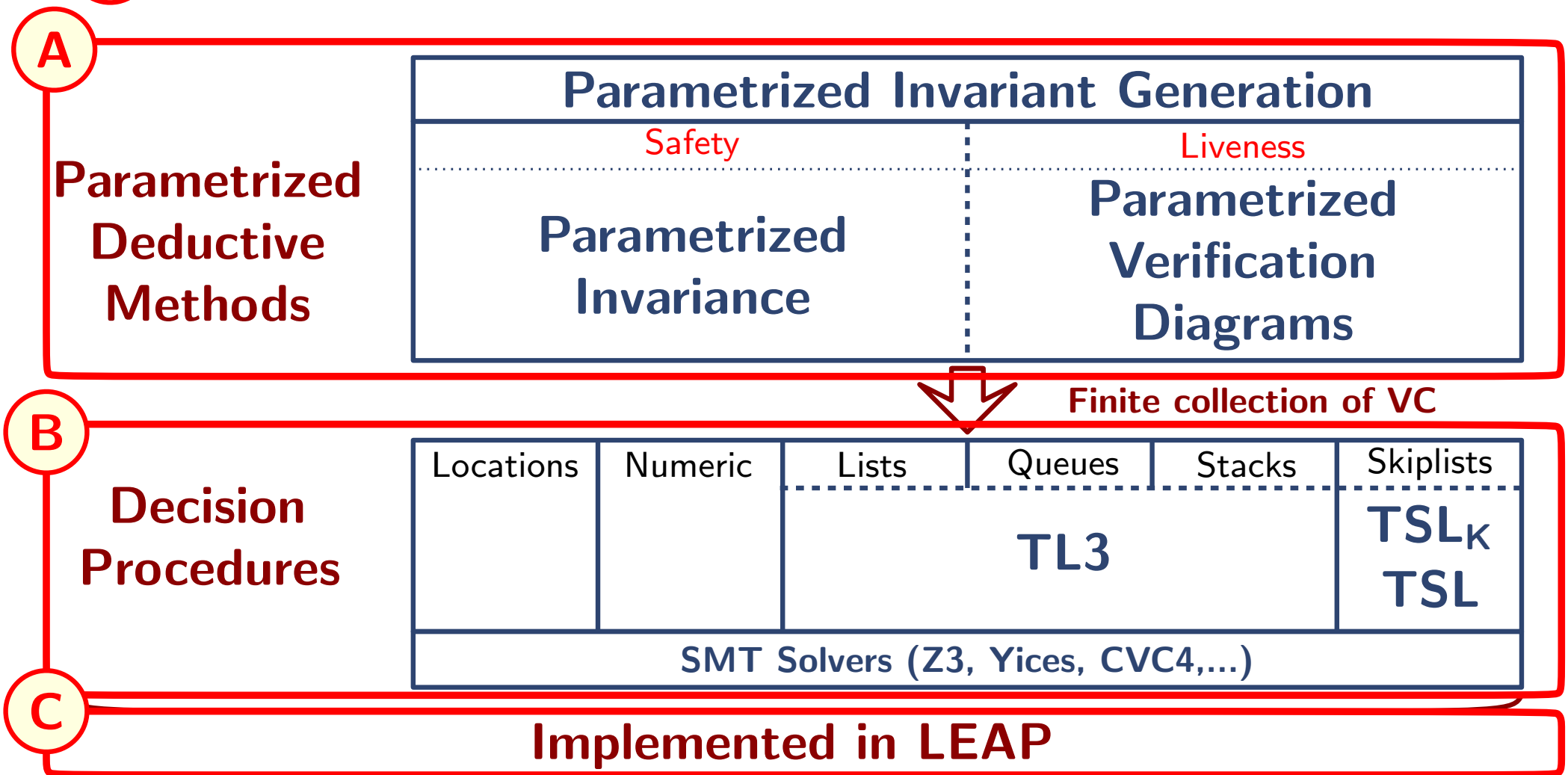


Our Contributions

A Deductive Verification Techniques for Parametrized Systems

B Decision Procedures for Complex Data Structures

C Implementation and Evaluation of our Framework



Our Contributions

A **Deductive Verification Techniques for Parametrized Systems**

B **Decision Procedures for Complex Data Structures**

C **Implementation and Evaluation of our Framework**

Our Contributions

A Deductive Verification Techniques for Parametrized Systems

1 Parametrized Invariance

Deductive proof rules for concurrent parametrized invariants

B Decision Procedures for Complex Data Structures

C Implementation and Evaluation of our Framework

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2 Parametrized Verification Diagrams
Diagram based verification for concurrent parametrized liveness properties

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Diagram based verification for concurrent parametrized liveness properties

3 Invariant Generation using Abstract Interpretation
Automatic parametrized invariant generation using off-the-shelf sequential absint

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B Decision Procedures for Complex Data Structures

4 TL3: A Decidable Theory for Concurrent Lists
A Theory and decision procedure for concurrent data structures of the shape of a list

5 TSL_K: A Decidable Family for Concurrent Bounded Skiplists
Theories and decision procedures for concurrent skiplists of at most K levels

6 TSL: A Decidable Theory for Skiplists with Arbitrary Levels
Theory and decision procedure for skiplists with unbounded many levels

C Implementation and Evaluation of our Framework

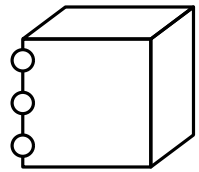
Verification of Concurrent Data-structures

Our verification approach

Verification of Concurrent Data-structures

Our verification approach

Concurrent DataStructure



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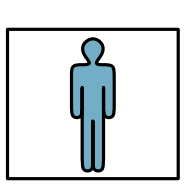

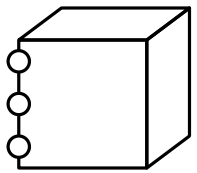

Most General Client

Verification of Concurrent Data-structures

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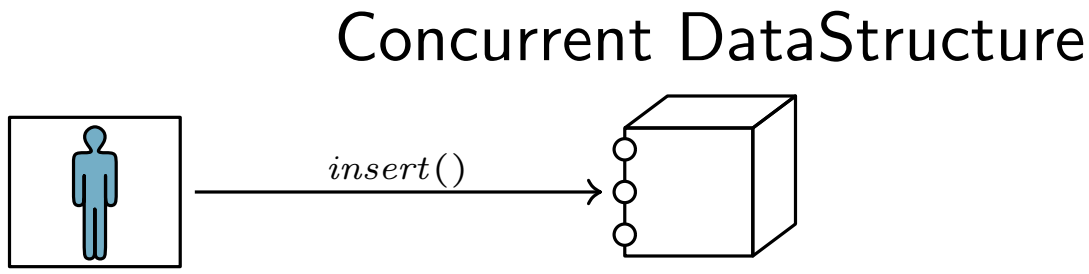

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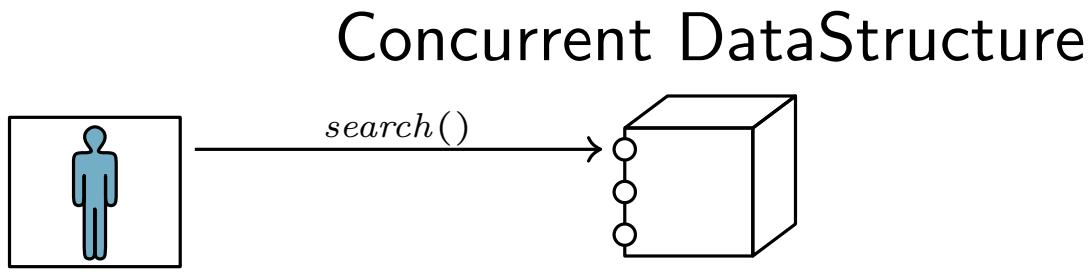

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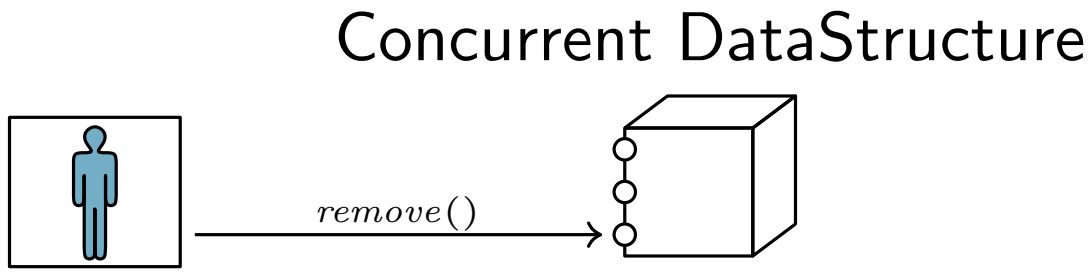

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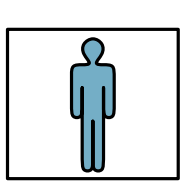

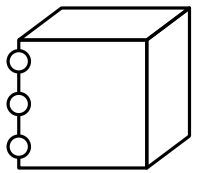

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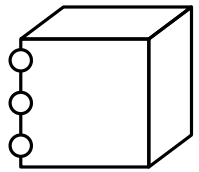

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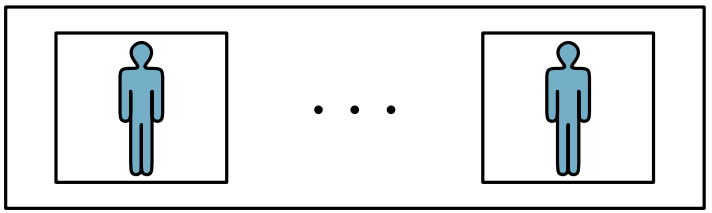

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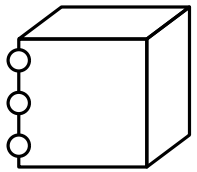

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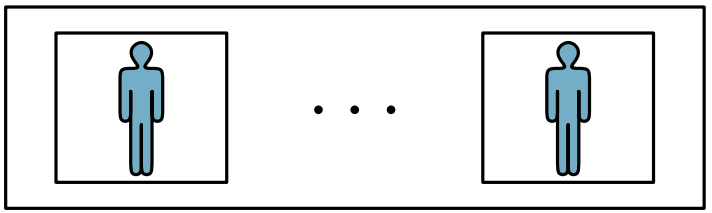

$$P[N] : P(1)||\cdots||P(N)$$

Verification of Concurrent Data-structures

Our verification approach

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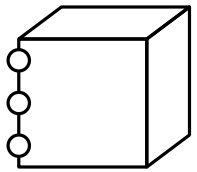

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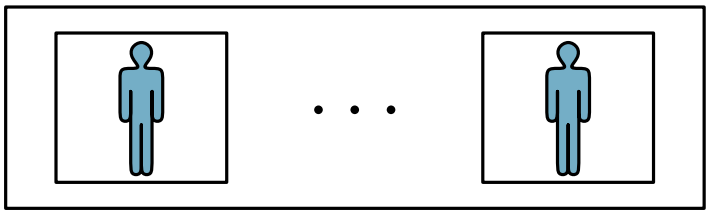

$P[N] : P(1)|| \cdots ||P(N)$

Property

φ

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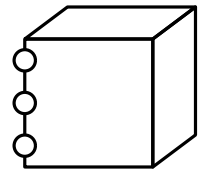

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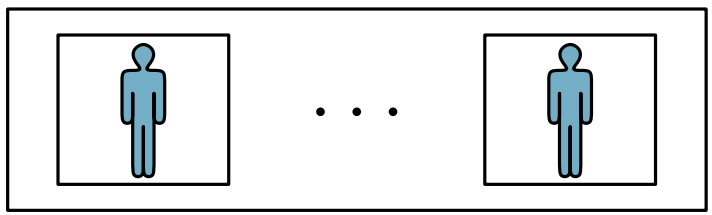

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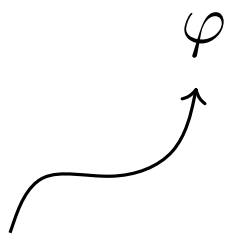
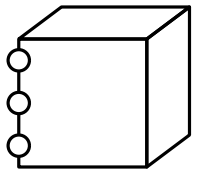


LTL ($\Box$,$\Diamond$,$\mathcal{U}$,. . .)

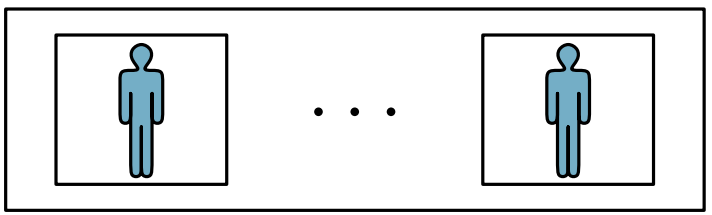
Verification of Concurrent Data-structures

Our verification approach

Concurrent DataStructure



Most General Client



$P[N] : P(1)||\cdots||P(N)$

Safety
Property

φ

LTL ($\Box$,$\Diamond$,$\mathcal{U}$,. . .)

Verification of Concurrent Data-structures

Our verification approach

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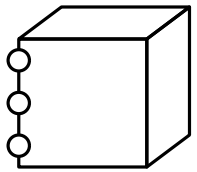

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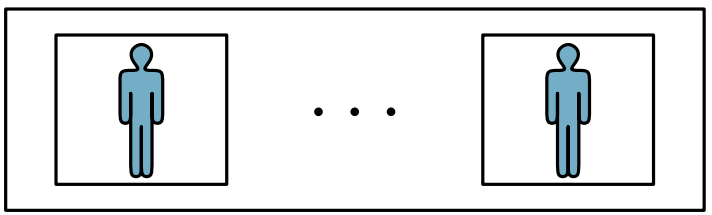

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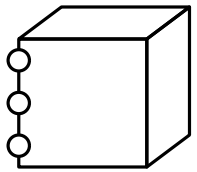

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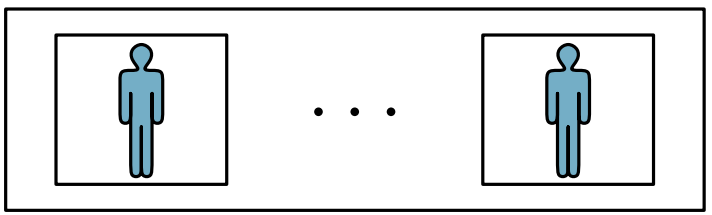

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$\models ?$

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Verification of Concurrent Data-structures

Safety Properties

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Parametrized Property

Verification of Concurrent Data-structures

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Uniform Verification Problem

Non-parametrized General Invariance Rule

To show that P satisfies $\Box p$, find q:

$$\frac{\begin{array}{lcl} \text{I1.} & \Theta \rightarrow q & \\ \text{I2.} & q \wedge \tau \rightarrow q' & \text{for all } \tau \\ \text{I3.} & q \rightarrow p & \end{array}}{\Box p}$$

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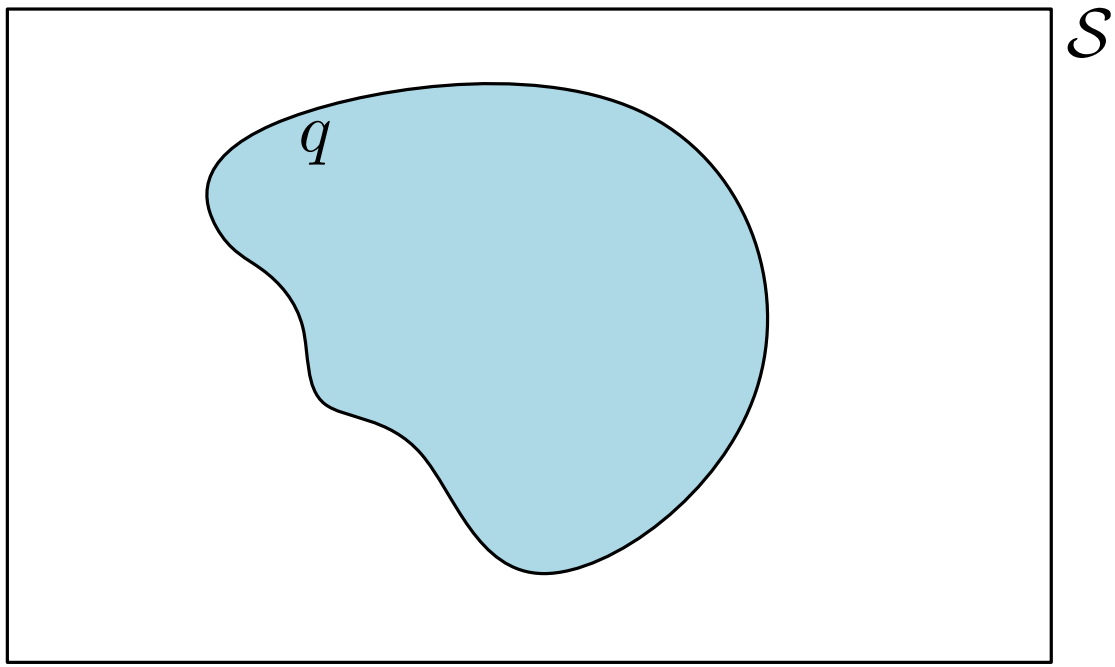

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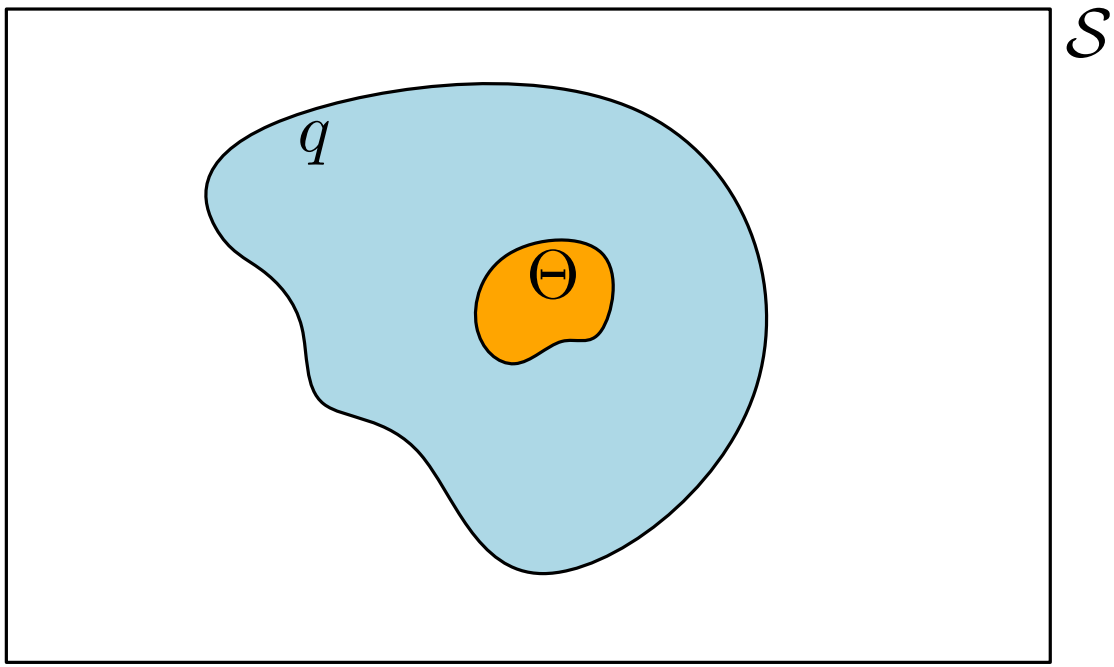

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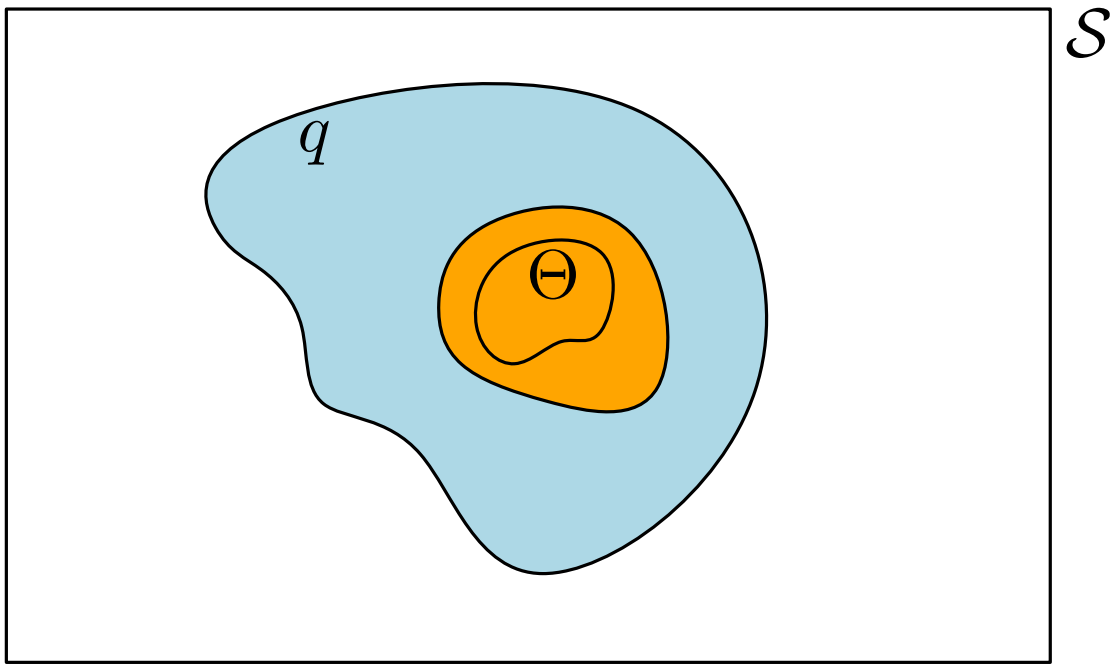

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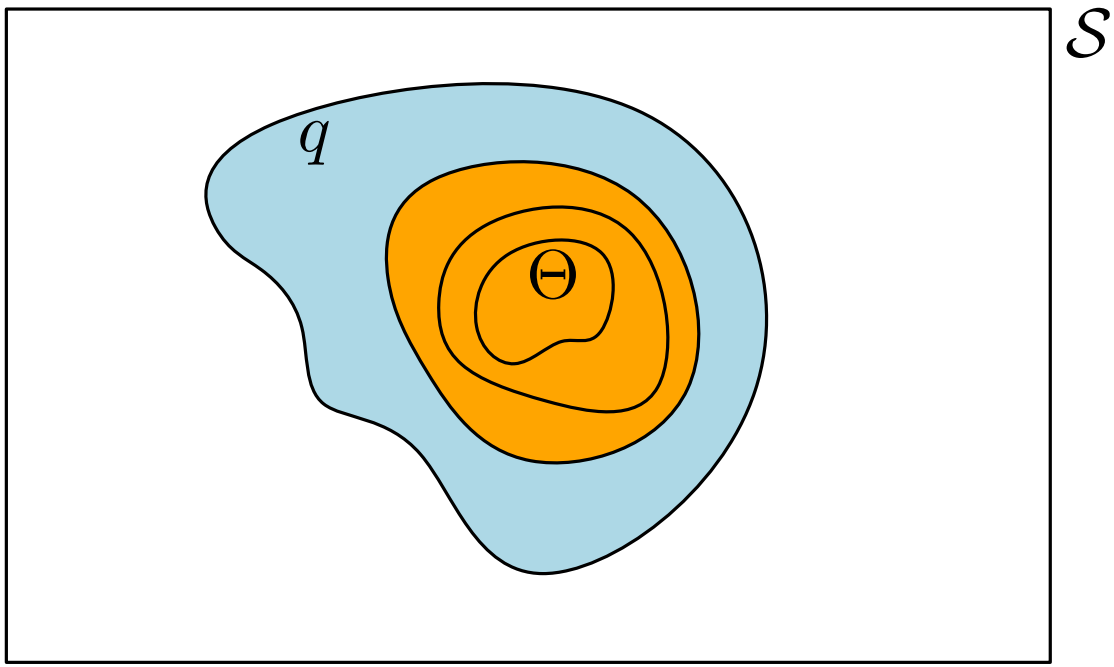

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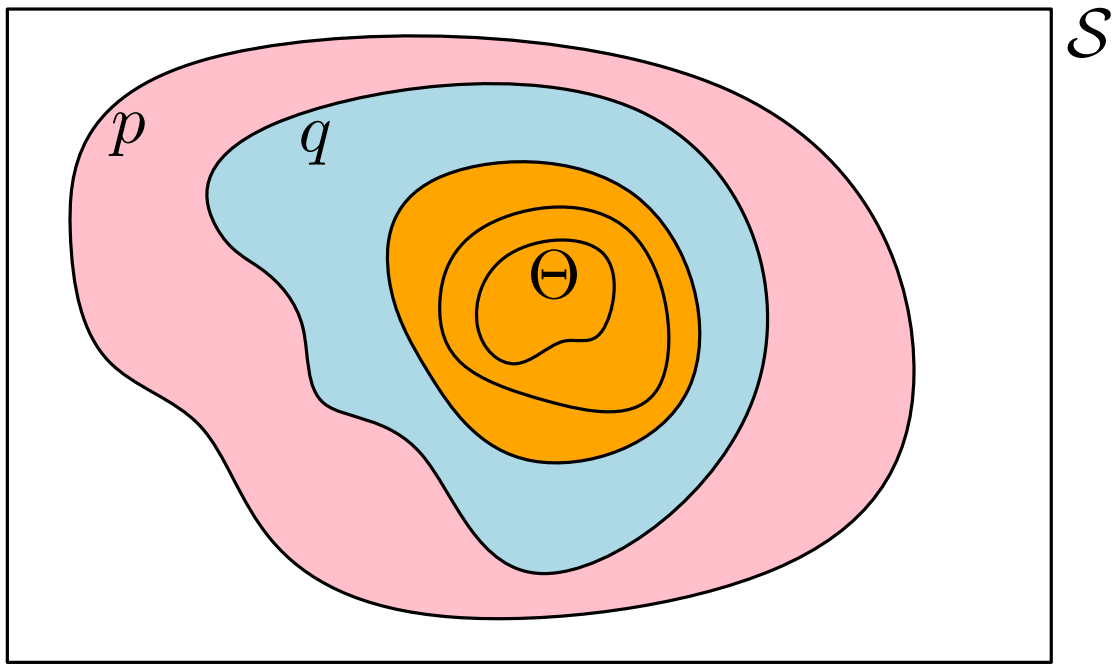

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Why this rule does not work for parametrized systems?

Motivating Example: Mutual Exclusion Protocol

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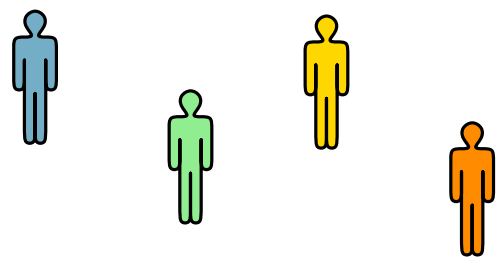

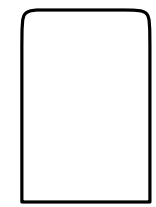

Critical Section

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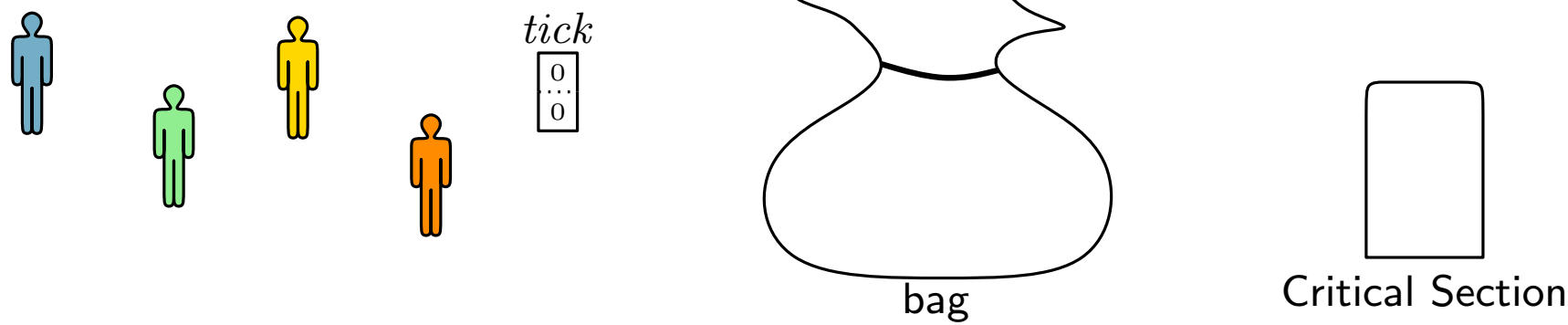

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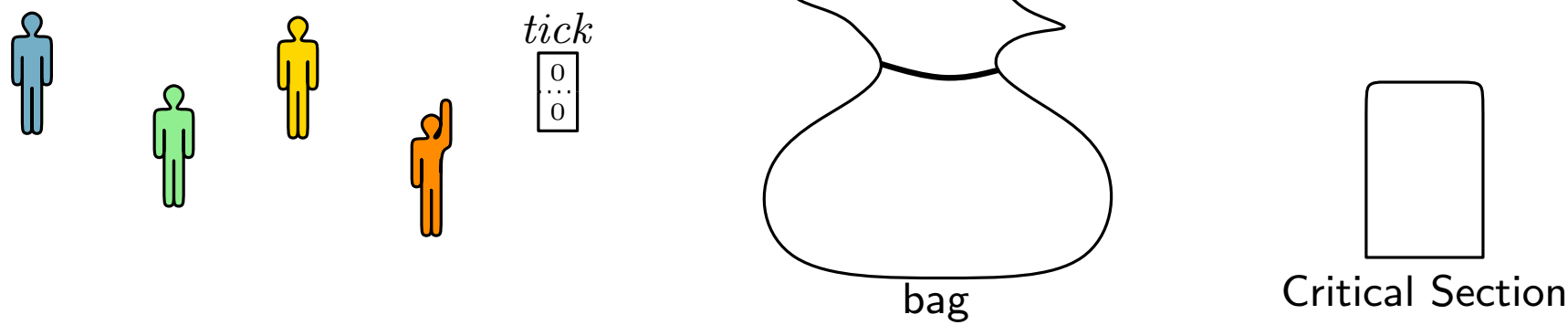

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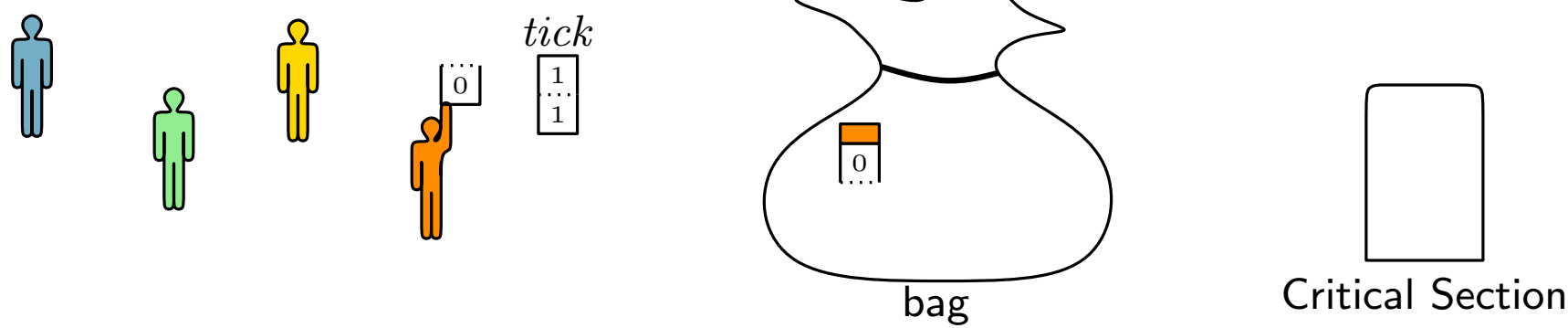

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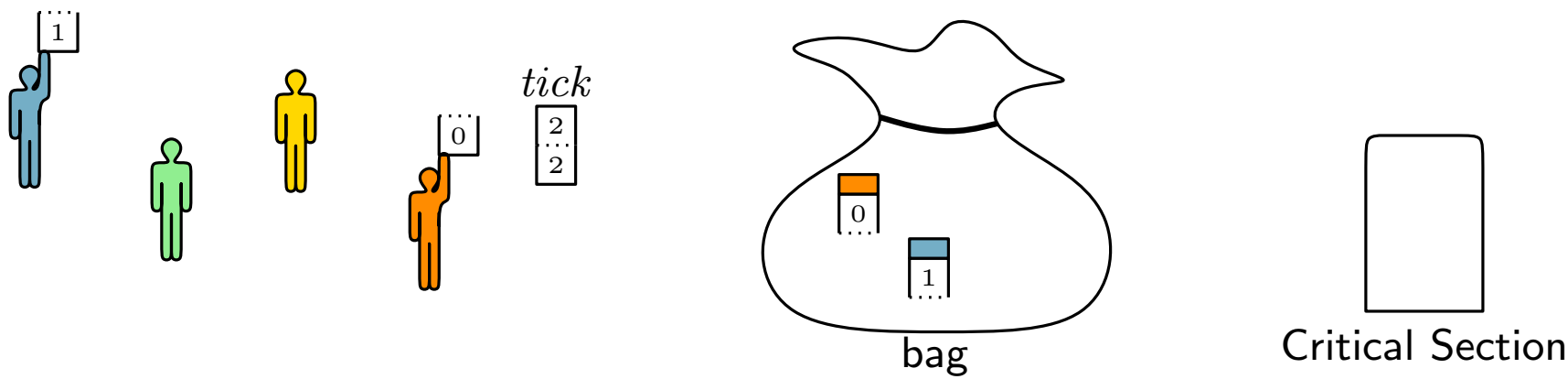

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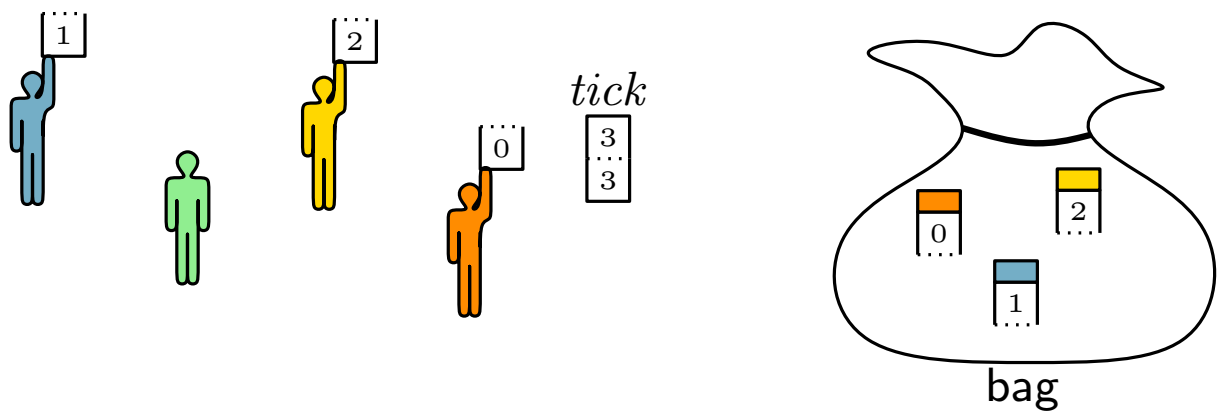

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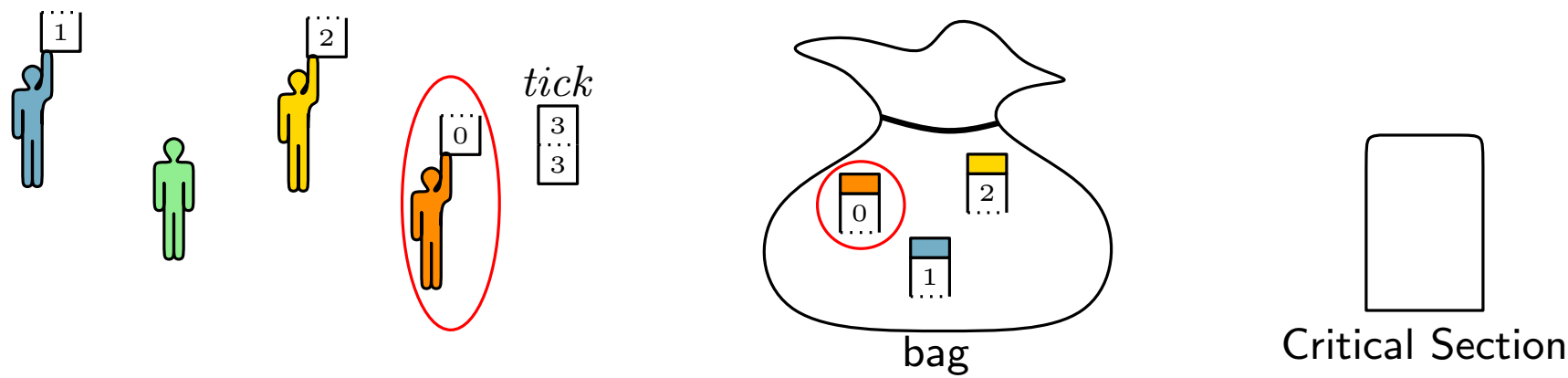

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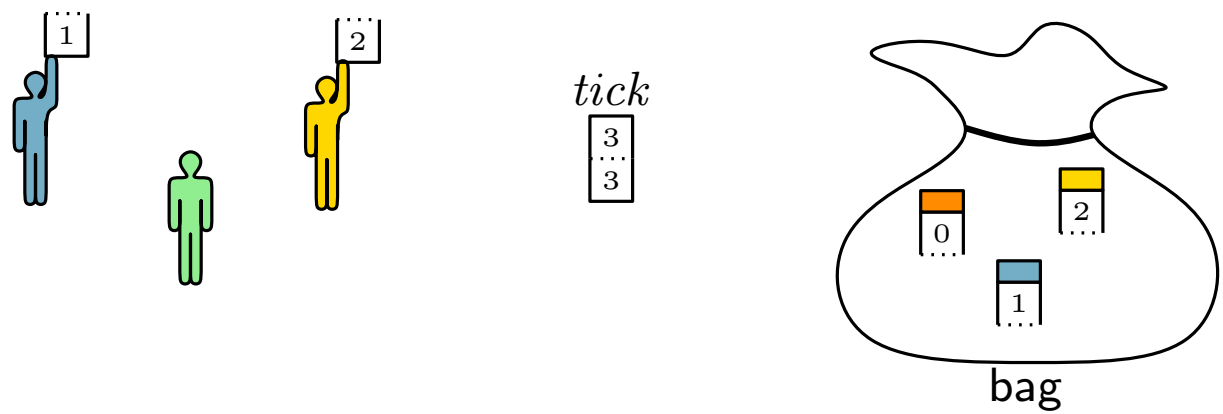

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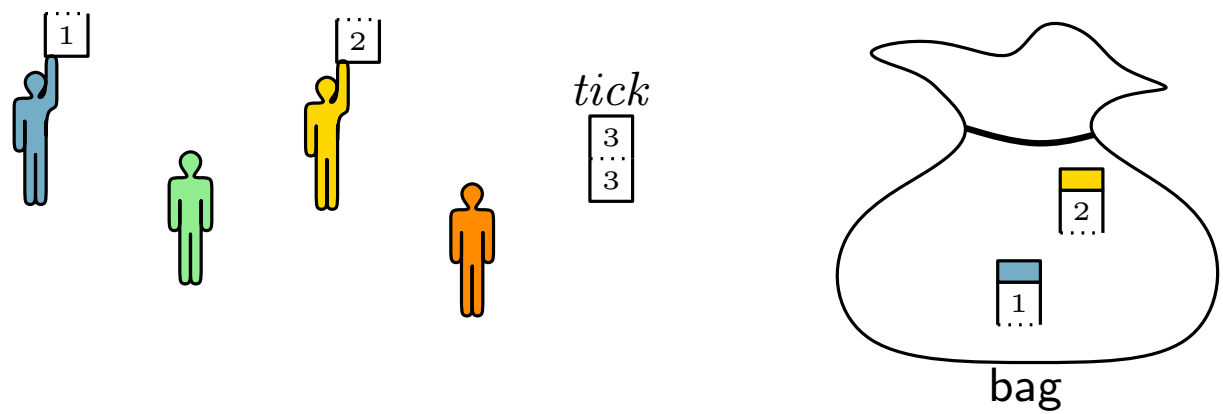

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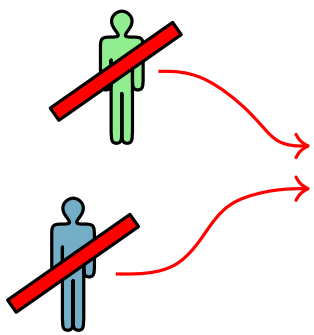

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$$\mathsf{mutex}(i,j) \mathrel{\hat{=}} \Box\left[i \neq j \rightarrow \neg(critical(i) \wedge critical(j))\right]$$

General Deductive Verification [Manna-Pnueli '95]

1.	Θ	$\rightarrow$	mutex	
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- Initiation: **1 VC**

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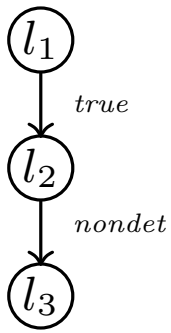

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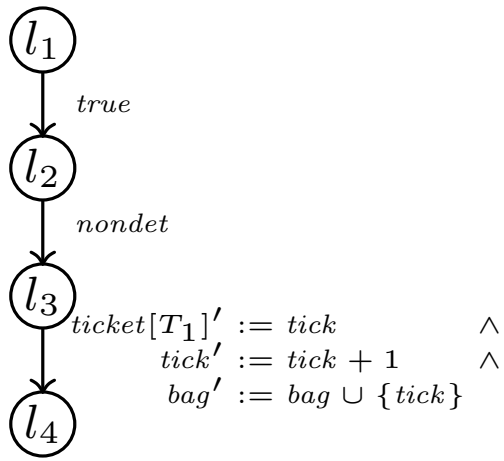

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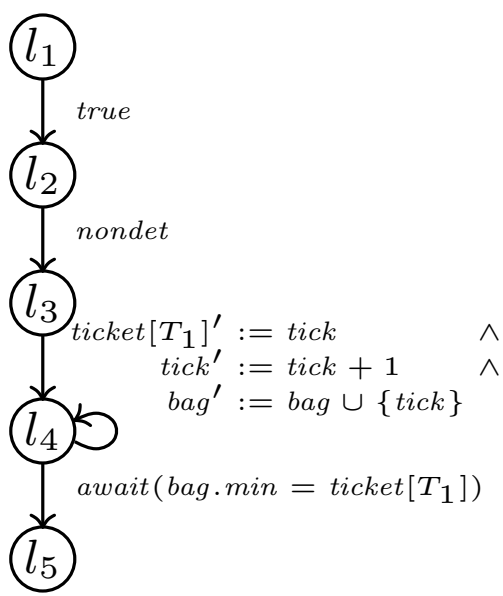

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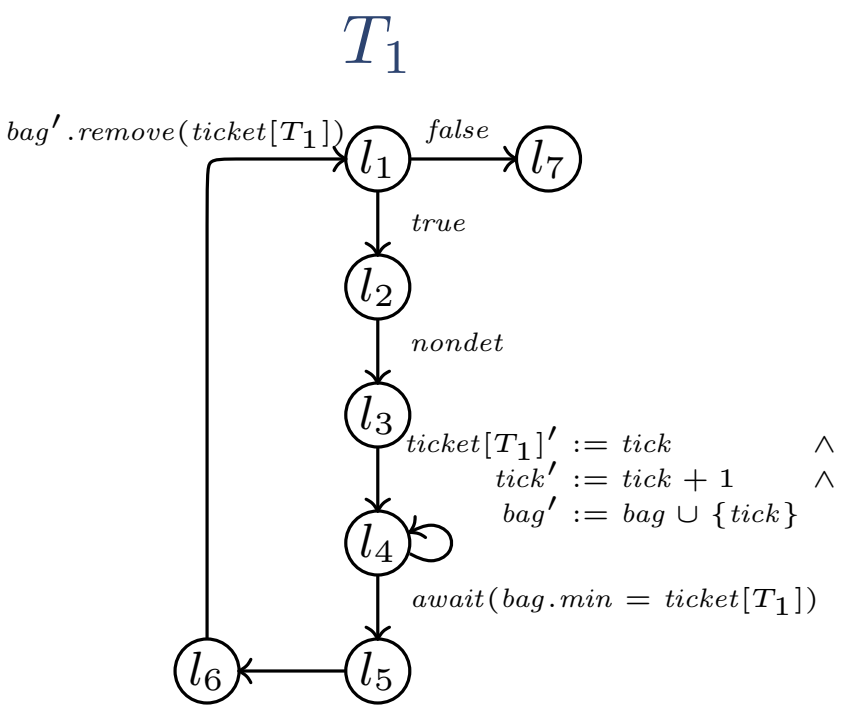

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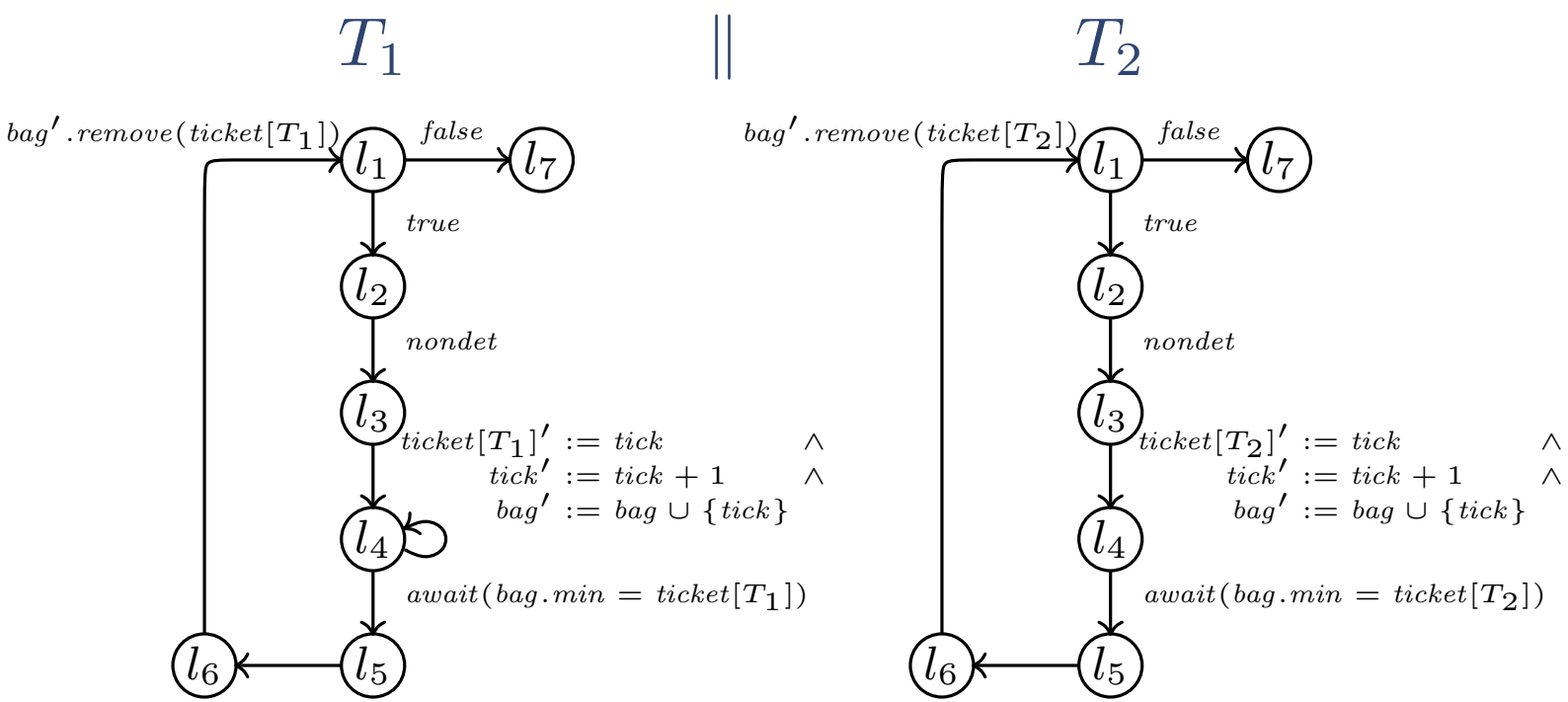

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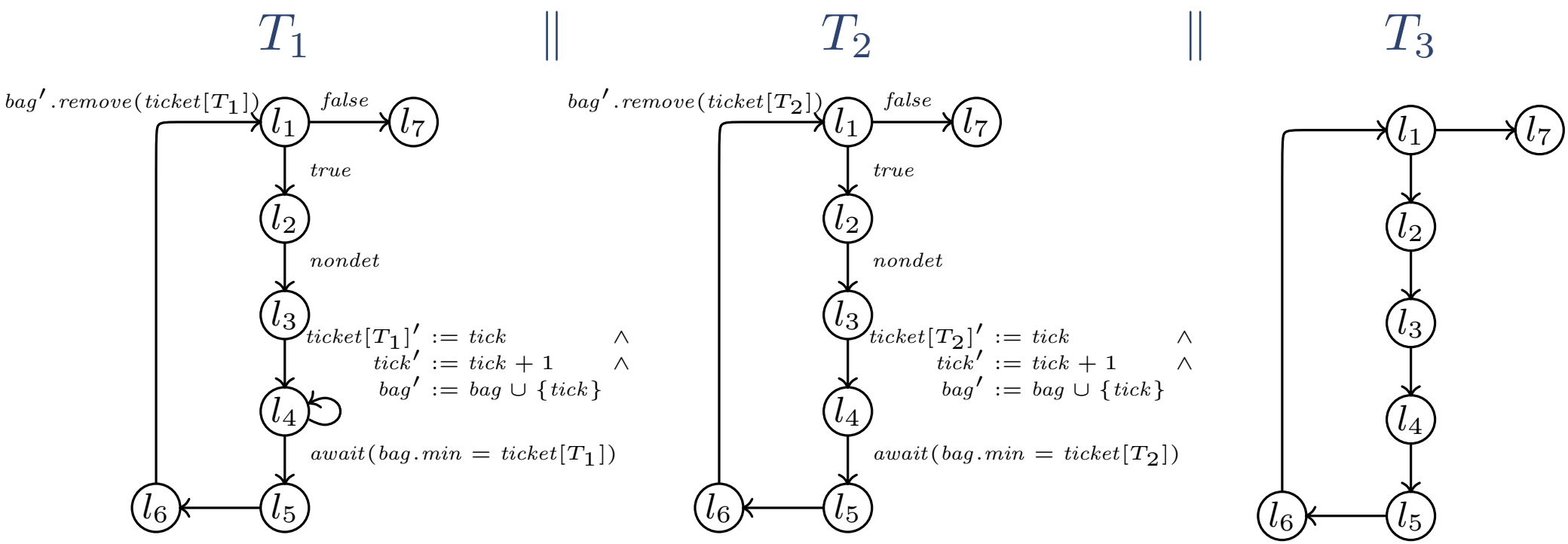
- Consecution: **16 VC**



General Deductive Verification [Manna-Pnueli '95]

1.		Θ	$\rightarrow$	mutex
2.		mutex $\wedge\ \tau$	$\rightarrow$	mutex$'$ for each τ

2 Threads T_1, T_2

- Initiation: **1 VC**

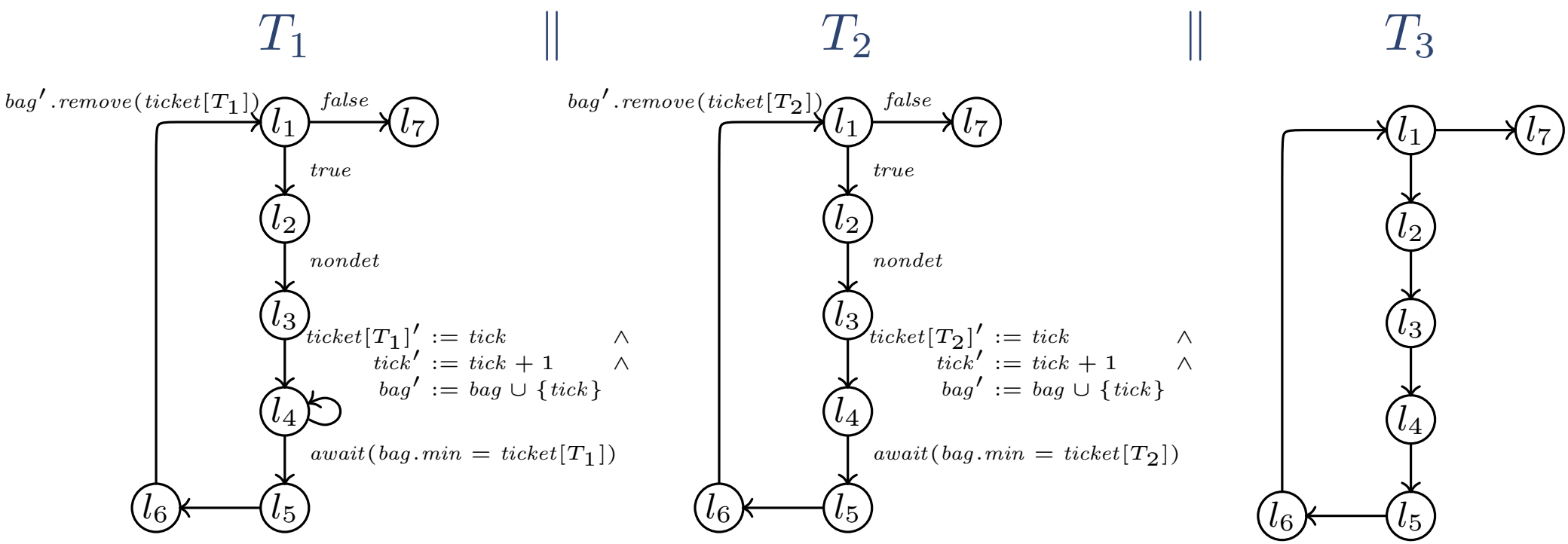
$\Theta_G : tick = 0 \wedge bag = \emptyset$

$\Theta_{T_1} : ticket[T_1] = 0 \wedge pc[T_1] = 1$

$\Theta_{T_2} : ticket[T_2] = 0 \wedge pc[T_2] = 1$

$\Theta_{T_3} : ticket[T_3] = 0 \wedge pc[T_3] = 1$

- Consecution: ~~**16 VC**~~ **24 VC**



Parametrized Invariance

Problem

Unbounded number of **verification conditions**

Parametrized Invariance

Problem

Unbounded number of **verification conditions**

Our solution

- **Specialized** invariance proof rules
- **Finite and bounded** number of verification conditions

Parametrized Invariance exploits the **similarities** of **fully symetric systems**

Parametrized Invariance (p-inv)

- **Bounded** number of VC, based on **program** and **specification**

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To show that $\mathcal{S}$ satisfies $\varphi(i)$:

$$\frac{\begin{array}{lrcll} (\ \mathsf{I}\ ) & \Theta & \rightarrow & \varphi & \\ (\mathsf{SC}) & \varphi \wedge \tau^{(i)} & \rightarrow & \varphi' & \text{forall } \tau \\ (\mathsf{OC}) & \varphi \wedge k \neq i \wedge \tau^{(k)} & \rightarrow & \varphi' & \text{forall } \tau, \text{fresh } k \end{array}}{\Box\varphi}$$

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To show that $\mathcal{S}$ satisfies $\varphi(i)$: **Initiation**

$$
\begin{array}{lrcll}
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\hline
 & \Box\varphi & & &
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Self-consecution

$$\frac{\begin{array}{lrcll} (\ \mathsf{I}\ ) & \Theta & \rightarrow & \varphi & \\ (\mathsf{SC}) & \varphi \wedge \tau^{(i)} & \rightarrow & \varphi' & \text{forall } \tau \\ (\mathsf{OC}) & \varphi \wedge k \neq i \wedge \tau^{(k)} & \rightarrow & \varphi' & \text{forall } \tau, \text{fresh } k \end{array}}{\Box\varphi}$$

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Other-consecution

Parametrized Invariance (p-inv)

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To show that $\mathcal{S}$ satisfies $\varphi(i)$:

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- For our example: mutex(i, j)

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- For our example: mutex(i, j) **#VC : 1**

$$(\ \mathsf{I}\ ) \qquad \Theta(i,j) \rightarrow \text{mutex}$$

Parametrized Invariance (p-inv)

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- For our example: mutex(i, j) **#VC : 1 + 16**

$$\begin{array}{lrcll} (\ \mathsf{I}\ ) & \Theta(i,j) & \rightarrow & \text{mutex} & \\ (\mathsf{SC}) & \text{mutex} \wedge \tau^{(i)} & \rightarrow & \text{mutex}' & \text{forall } \tau \\ & \text{mutex} \wedge \tau^{(j)} & \rightarrow & \text{mutex}' & \text{forall } \tau \end{array}$$

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- For our example: mutex(i, j) **#VC : 1 + 16 + 8 = 25**

$$\begin{array}{lrcll} (\ \mathsf{I}\ ) & \Theta(i,j) & \rightarrow & \text{mutex} & \\ (\mathsf{SC}) & \text{mutex} \wedge \tau^{(i)} & \rightarrow & \text{mutex}' & \text{forall } \tau \\ & \text{mutex} \wedge \tau^{(j)} & \rightarrow & \text{mutex}' & \text{forall } \tau \\ (\mathsf{OC}) & \text{mutex} \wedge k \neq i \wedge k \neq j \wedge \tau^{(k)} & \rightarrow & \text{mutex}' & \text{forall } \tau, \text{fresh } k \end{array}$$

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Independently on #threads in the system

- For our example: mutex(i, j) **#VC : 1 + 16 + 8 = 25**

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Parametrized Invariance (p-inv) is not enough

- Lets try to prove mutex using P-INV...

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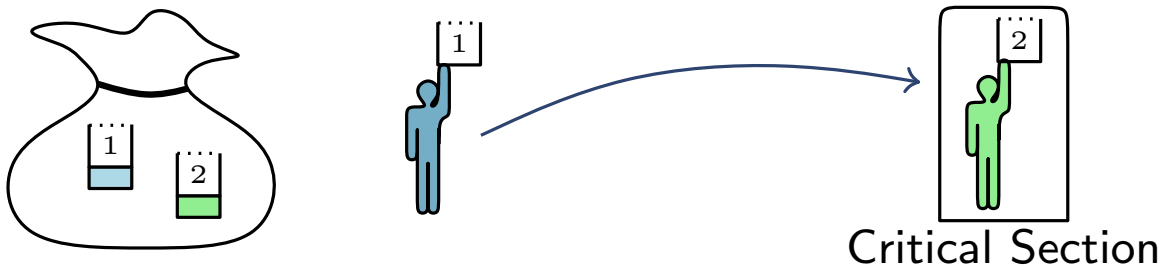
- ... transition 4 (i.e., **await** $(bag.min == ticket)$) **fails**



Because mutex does not encode that the
thread in the critical section owns the minimum ticket

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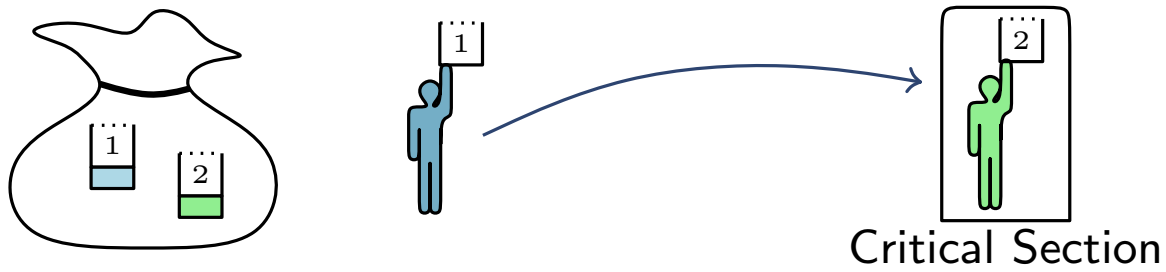

Because mutex does not encode that the
thread in the critical section owns the minimum ticket

- Extra support is required

$$\text{minticket}(i) \;\hat{=}\; \Box\big[critical(i) \rightarrow min(bag) = ticket(i)\big]$$

$$\text{notsame}(i,j) \;\hat{=}\; \Box\big[i \neq j \wedge active(i) \wedge active(j) \rightarrow ticket(i) \neq ticket(j)\big]$$

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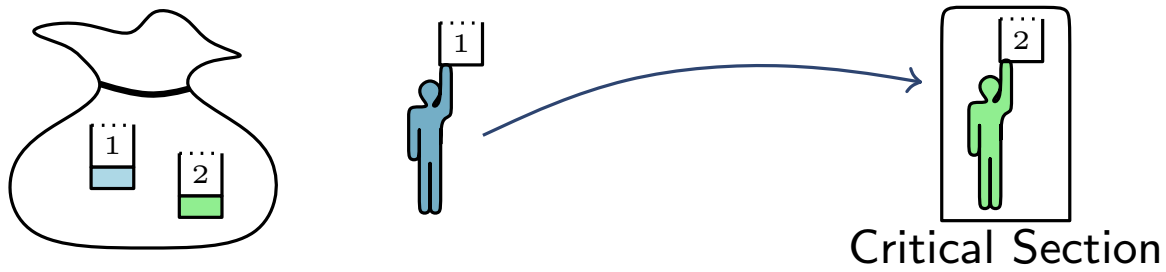

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We now require a **new rule** for **invariant support**

Parametrized Invariance with Support (sp-inv)

To show that $\mathcal{S}$ satisfies $\Box\varphi(i)$. Find $\psi(\overline{w})$ with:

$$
\begin{array}{lllllll}
(\ \mathsf{S}\ ) & & & & \Box\psi & & \\
(\ \mathsf{I}\ ) & & & \Theta & \rightarrow & \varphi & \\
(\mathsf{SC}) & \psi, \varphi \rhd & & \tau^{(i)} & \rightarrow & \varphi' & \text{forall } \tau \\
(\mathsf{OC}) & \psi, \varphi \rhd & k \neq i \wedge \tau^{(k)} & & \rightarrow & \varphi' & \text{forall } \tau\text{, fresh } k \\
\hline
 & & & \Box\varphi & & &
\end{array}
$$

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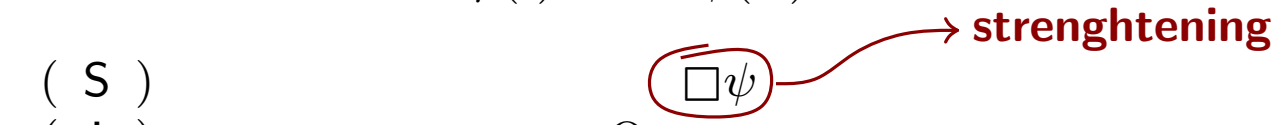

$$
\begin{array}{lllcll}
(\ \mathsf{S}\ ) & & & \Box\psi & & \\
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\hline
 & & \Box\varphi & & &
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$$

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To show that $\mathcal{S}$ satisfies $\Box\varphi(i)$. Find $\psi(\overline{w})$ with:

$$
\begin{array}{lllcll}
(\ \mathsf{S}\ ) & & & & \Box\psi & \text{strenghtening} \\
(\ \mathsf{I}\ ) & & \Theta & \rightarrow & \varphi & \text{initiation} \\
(\mathsf{SC}) & \psi, \varphi \rhd & \tau^{(i)} & \rightarrow & \varphi' & \text{self-consecution} \quad \text{forall } \tau \\
(\mathsf{OC}) & \psi, \varphi \rhd & k \neq i \wedge \tau^{(k)} & \rightarrow & \varphi' & \text{other-consecution} \quad \text{forall } \tau, \text{ fresh } k \\
\hline
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\end{array}
$$

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(\mathsf{OC}) & \psi, \varphi \rhd & k \neq i \wedge \tau^{(k)} & \to & \varphi' & \text{forall } \tau, \text{ fresh } k
\end{array}}{\Box\varphi}$$

Instantiate the assumptions for **self** and **others**

$\psi \rhd (A \to B)$ whether $\left[\left(\bigwedge_{\sigma \in S} \psi_\sigma \wedge A\right) \to B\right]$

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- **Example:** minticket and notsame to support mutex(i, j)

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(S) $\Box$minticket $\wedge$ $\Box$notsame

Parametrized Invariance with Support (sp-inv)

To show that $\mathcal{S}$ satisfies $\Box\varphi(i)$. Find $\psi(\overline{w})$ with:

(S)			$\Box\psi$		
(I)		Θ	$\rightarrow$	φ	
(SC)	$\psi, \varphi \rhd$	$\tau^{(i)}$	$\rightarrow$	φ'	forall τ
(OC)	$\psi, \varphi \rhd$	$k \neq i \wedge \tau^{(k)}$	$\rightarrow$	φ'	forall τ, fresh k
		$\Box\varphi$			

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(S) $\Box$minticket $\wedge$ $\Box$notsame

(I) $\Theta(i, j) \rightarrow$ mutex

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$$
\begin{array}{ll}
(\ \mathsf{S}\ ) & \Box\mathsf{minticket} \wedge \Box\mathsf{notsame} \\
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(\mathsf{SC}) & \left[\bigwedge_{\sigma \in \{t_1,\ldots,t_5\} \rightarrow \{i,j\}} \left[\begin{array}{l} \mathsf{minticket}(t_1) \ \wedge \\ \mathsf{notsame}(t_2, t_3) \wedge \\ \mathsf{mutex}(t_4, t_5) \end{array} \right]_\sigma \wedge \tau^{(i)} \right] \rightarrow \mathsf{mutex}' \quad \forall\tau
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To show that $\mathcal{S}$ satisfies $\Box\varphi(i)$. Find $\psi(\overline{w})$ with:

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(\ \mathsf{S}\ ) & \Box\mathsf{minticket} \wedge \Box\mathsf{notsame} & \\
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(\mathsf{SC}) & \left[\bigwedge_{\sigma\in\{t_1,\ldots,t_5\}\rightharpoonup\{i,j\}} \left[\begin{array}{l}\mathsf{minticket}(t_1)\ \wedge \\ \mathsf{notsame}(t_2,t_3)\wedge \\ \mathsf{mutex}(t_4,t_5)\end{array}\right]_\sigma \wedge \tau^{(i)}\right] \rightarrow \mathsf{mutex}' \quad \forall\tau & \\
 & \left[\bigwedge_{\sigma\in\{t_1,\ldots,t_5\}\rightharpoonup\{i,j\}} \left[\begin{array}{l}\mathsf{minticket}(t_1)\ \wedge \\ \mathsf{notsame}(t_2,t_3)\wedge \\ \mathsf{mutex}(t_4,t_5)\end{array}\right]_\sigma \wedge \tau^{(j)}\right] \rightarrow \mathsf{mutex}' \quad \forall\tau &
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\begin{array}{lllll}
(\ \mathsf{S}\ ) & \Box\text{minticket} \wedge \Box\text{notsame} & & & \\
(\ \mathsf{I}\ ) & & \Theta(i,j) & \rightarrow & \text{mutex} \\
(\mathsf{SC}) & \left[\bigwedge_{\sigma \in \{t_1,\ldots,t_5\} \rightarrow \{i,j\}} \left[\begin{array}{l} \text{minticket}(t_1) \ \wedge \\ \text{notsame}(t_2,t_3) \wedge \\ \text{mutex}(t_4,t_5) \end{array} \right]_\sigma \wedge \tau^{(i)} \right] & \rightarrow & \text{mutex}' & \forall\tau \\
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(\mathsf{OC}) & \left[\bigwedge_{\sigma \in \{t_1,\ldots t_5\} \rightarrow \{i,j,k\}} \left[\begin{array}{l} \text{minticket}(t_1) \ \wedge \\ \text{notsame}(t_2,t_3) \wedge \\ \text{mutex}(t_4,t_5) \end{array} \right]_\sigma \wedge k \neq i \wedge k \neq j \wedge \tau^{(k)} \right] & \rightarrow & \text{mutex}' & \forall\tau
\end{array}
$$

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To show that $\mathcal{S}$ satisfies $\Box\varphi(i)$. Find $\psi(\overline{w})$ with:

$$\frac{\begin{array}{llrcll}
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(\mathsf{SC}) & \psi, \varphi \rhd & \tau^{(i)} & \to & \varphi' & \text{forall } \tau \\
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\end{array}}{\Box\varphi}$$

Instantiate the assumptions for **self** and **others**

Problem

Sometimes we have **invariant circular dependency**

Parametrized Invariance with Graph Support (g-inv)

To show that $\mathcal{S}$ satisfies $\Box\varphi_i(\overline{v}) \wedge \Box\varphi_j(\overline{w})$:

$$
\begin{array}{lllll}
(\ \mathsf{I}\ ) & & \Theta & \rightarrow & \varphi_i \wedge \varphi_j \\
(\ \mathsf{SC_i}\ ) & \varphi_i, \varphi_j \rhd & \tau^{(t)} & \rightarrow & \varphi_i' \qquad \text{forall } \tau, \text{ forall } t \in \overline{v} \\
(\ \mathsf{SC_j}\ ) & \varphi_i, \varphi_j \rhd & \tau^{(t)} & \rightarrow & \varphi_j' \qquad \text{forall } \tau, \text{ forall } t \in \overline{w} \\
(\mathsf{OC_i}) & \varphi_i, \varphi_j \rhd & \bigwedge_{x \in \overline{v}} k \neq x \wedge \tau^{(k)} & \rightarrow & \varphi_i' \qquad \text{forall } \tau, \text{ fresh } k \notin \overline{v} \\
(\mathsf{OC_j}) & \varphi_i, \varphi_j \rhd & \bigwedge_{x \in \overline{w}} k \neq x \wedge \tau^{(k)} & \rightarrow & \varphi_j' \qquad \text{forall } \tau, \text{ fresh } k \notin \overline{w} \\
\hline
 & & \Box\varphi_i \wedge \Box\varphi_j & &
\end{array}
$$

Parametrized Invariance with Graph Support (g-inv)

To show that $\mathcal{S}$ satisfies $\Box\varphi_i(\overline{v}) \wedge \Box\varphi_j(\overline{w})$:

initiation

(I)		Θ	$\rightarrow$	$\varphi_i \wedge \varphi_j$	
(SC$_\text{i}$)	$\varphi_i, \varphi_j \rhd$	$\tau^{(t)}$	$\rightarrow$	φ_i'	forall τ, forall $t \in \overline{v}$
(SC$_\text{j}$)	$\varphi_i, \varphi_j \rhd$	$\tau^{(t)}$	$\rightarrow$	φ_j'	forall τ, forall $t \in \overline{w}$
(OC$_\text{i}$)	$\varphi_i, \varphi_j \rhd$	$\bigwedge_{x \in \overline{v}} k \neq x \wedge \tau^{(k)}$	$\rightarrow$	φ_i'	forall τ, fresh $k \notin \overline{v}$
(OC$_\text{j}$)	$\varphi_i, \varphi_j \rhd$	$\bigwedge_{x \in \overline{w}} k \neq x \wedge \tau^{(k)}$	$\rightarrow$	φ_j'	forall τ, fresh $k \notin \overline{w}$

$\Box\varphi_i \wedge \Box\varphi_j$

Parametrized Invariance with Graph Support (g-inv)

To show that $\mathcal{S}$ satisfies $\Box\varphi_i(\overline{v}) \wedge \Box\varphi_j(\overline{w})$:

initiation

self-consecution

$$
\begin{array}{lllll}
(\ \mathsf{I}\ ) & & \Theta & \rightarrow & \varphi_i \wedge \varphi_j \\
(\ \mathsf{SC_i}\ ) & \varphi_i, \varphi_j \rhd & \tau^{(t)} & \rightarrow & \varphi_i' \qquad \text{forall } \tau, \text{ forall } t \in \overline{v} \\
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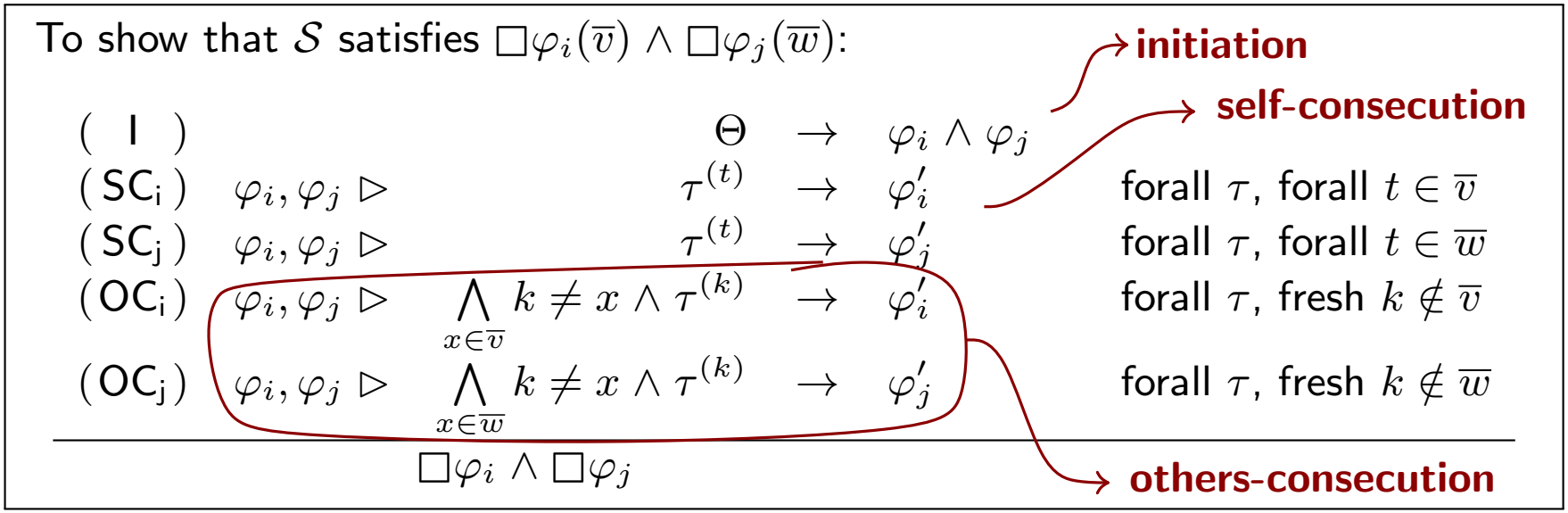
\end{array}
$$

$$\Box\varphi_i \wedge \Box\varphi_j$$

Parametrized Invariance with Graph Support (g-inv)



Parametrized Invariance with Graph Support (g-inv)

To show that $\mathcal{S}$ satisfies $\Box\varphi_i(\overline{v}) \wedge \Box\varphi_j(\overline{w})$:

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(SC$_\text{i}$)	$\varphi_i, \varphi_j \triangleright$	$\tau^{(t)}$	$\rightarrow$	φ_i'	forall τ, forall $t \in \overline{v}$
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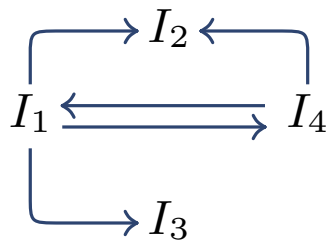
		$\Box\varphi_i \wedge \Box\varphi_j$			

initiation

self-consecution

others-consecution

- A **generalization** of G-INV is a **proof graph**



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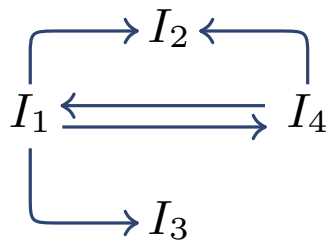

Theorem

Every node is invariant if every node is either:

- an inductive invariant, or
- has an incident edge and all VCs are valid

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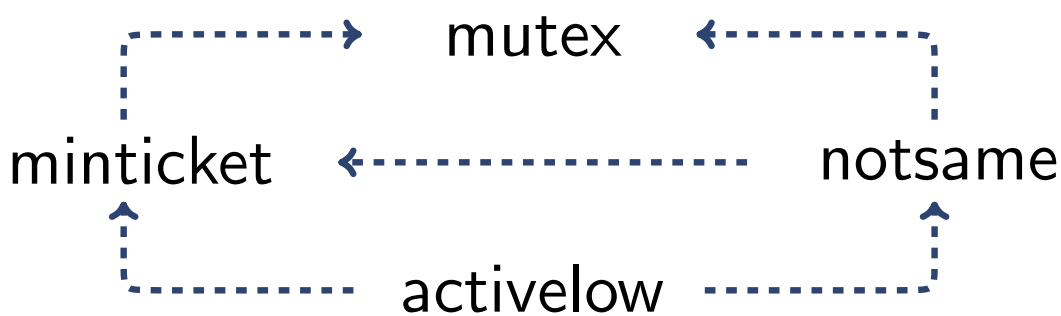

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Our Contributions

That's for safety... what about **liveness**?

Our Contributions

A Deductive Verification Techniques for Parametrized Systems

1 Parametrized Invariance
Deductive proof rules for concurrent parametrized invariants

2 Parametrized Verification Diagrams
Diagram based verification for concurrent parametrized liveness properties

3 Invariant Generation with Self-refelction
Automatic parametrized invariant generation using off-the-shelf sequential absint

B Decision Procedures for Complex Data Structures

4 TL3: A Decidable Theory for Concurrent Lists
A Theory and decision procedure for concurrent data structures of the shape of a list

5 TSL_K: A Decidable Family for Concurrent Bounded Skiplists
Theories and decision procedures for concurrent skiplists of at most K levels

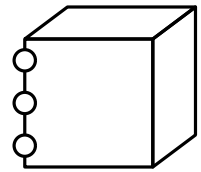
6 TSL: A Decidable Theory for Skiplists with Arbitrary Levels
Theory and decision procedure for skiplists with unbounded many levels

C Implementation and Evaluation of our Framework

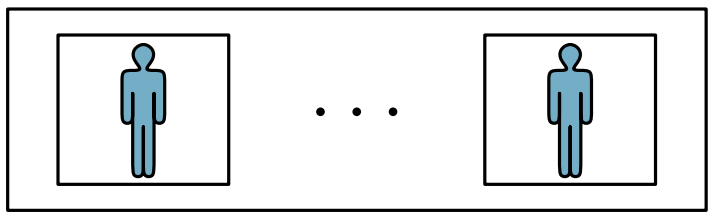
Verification of Concurrent Data-structures

Our verification approach

Concurrent DataStructure

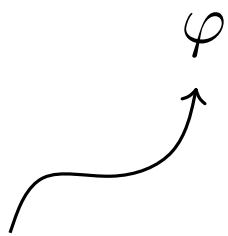


Most General Client



$P[N] : P(1)||\cdots||P(N)$

Property



LTL ($\Box$,$\Diamond$,$\mathcal{U}$,. . .)

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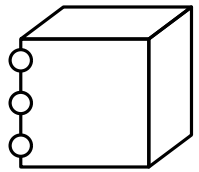
Our verification approach

Concurrent DataStructure



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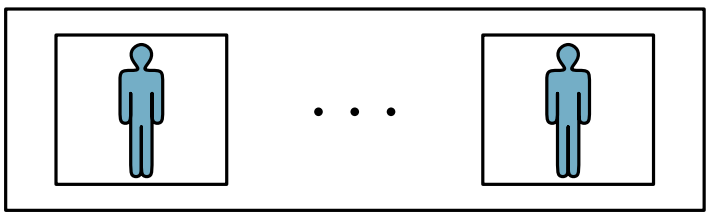
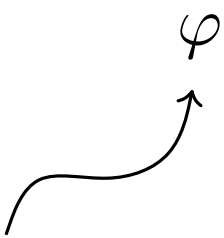


$P[N] : P(1)||\cdots||P(N)$

Liveness
Property



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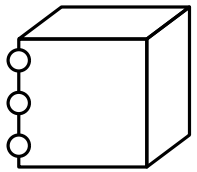

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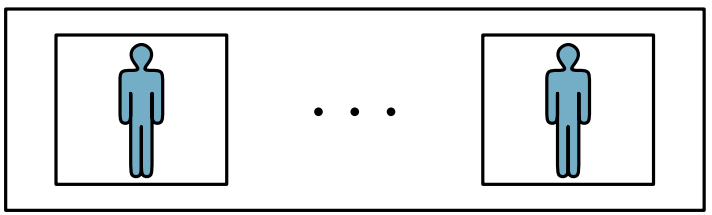

$P[N] : P(1)||\cdots||P(N)$

Liveness Property

$\varphi^{(k)}$

Verification of Concurrent Data-structures

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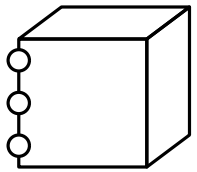

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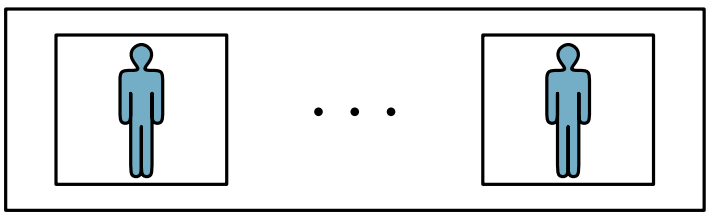


$P[N] : P(1)||\cdots||P(N)$

$\models$?

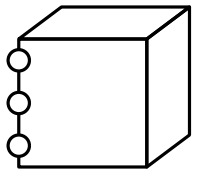
Liveness Property

$\varphi^{(k)}$

Verification of Concurrent Data-structures

Our verification approach

Concurrent DataStructure



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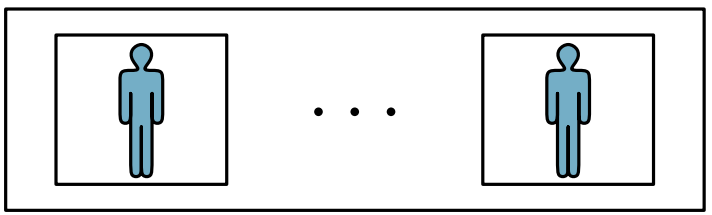


$P[N] : P(1)|| \cdots ||P(N)$

Diagram

$\mathcal{D}$

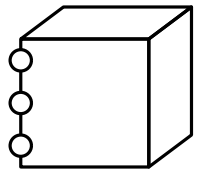
Liveness Property

$\varphi^{(k)}$

Verification of Concurrent Data-structures

Our verification approach

Concurrent DataStructure



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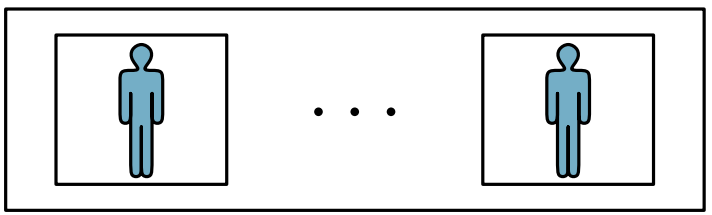


$P[N] : P(1)||\cdots||P(N)$

Diagram

$\mathcal{D}$

Liveness Property

$\varphi^{(k)}$

$\models$ Verification Conditions:

- Initiation
- Consecution
- Acceptance
- Fairness

$\models$ Satisfaction (Model Checking)

The Need of Parametrized Diagrams

System: $\textsc{SetMutex}(T_1)||\textsc{SetMutex}(T_2)$

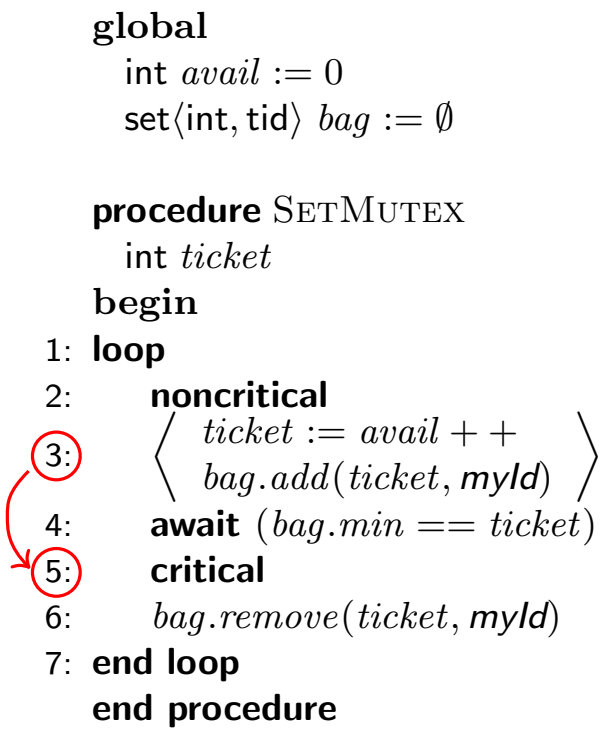
global
  int $avail := 0$
  set⟨int, tid⟩ $bag := \emptyset$

procedure SetMutex
  int $ticket$
begin
1: **loop**
2: **noncritical**
3: $\left\langle \begin{array}{l} ticket := avail++ \\ bag.add(ticket, myId) \end{array} \right\rangle$
4: **await** $(bag.min == ticket)$
5: **critical**
6: $bag.remove(ticket, myId)$
7: **end loop**
end procedure

The Need of Parametrized Diagrams

System: $\textsc{SetMutex}(T_1)||\textsc{SetMutex}(T_2)$

Spec: $\Box\left(pc_{T_1} = 3 \rightarrow \Diamond pc_{T_1} = 5\right)$



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T_1 not interested

```
   global
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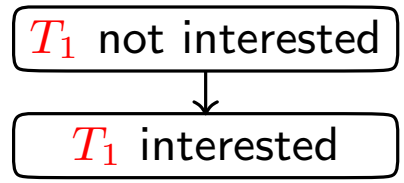


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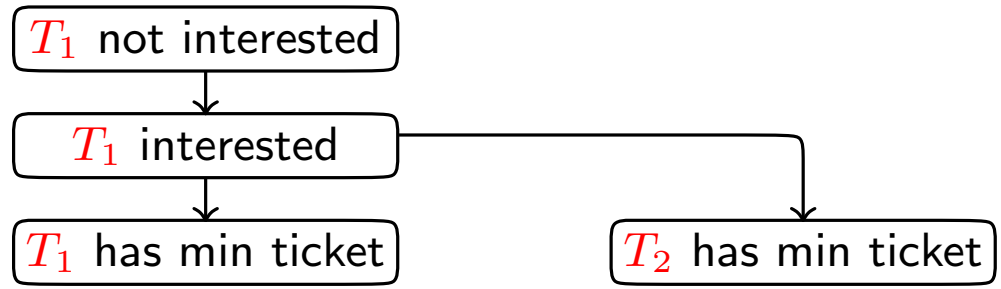


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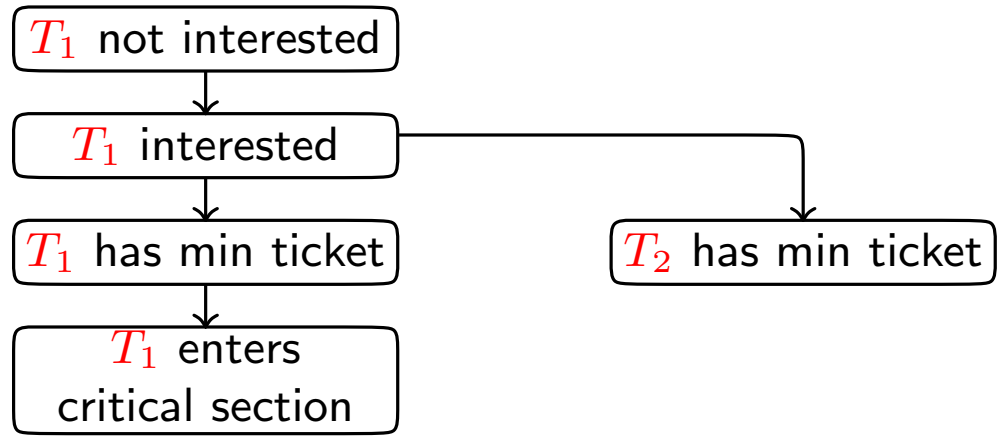

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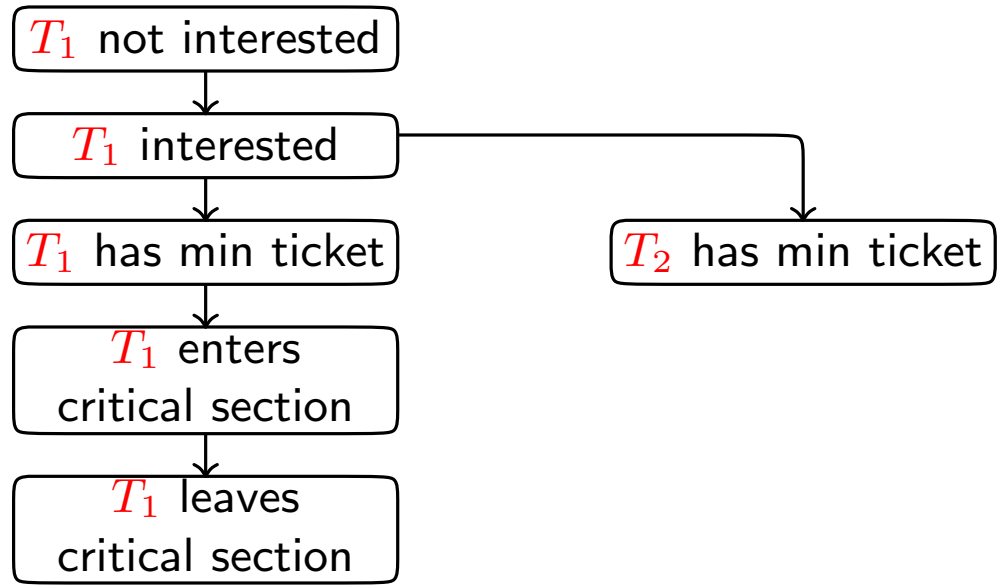

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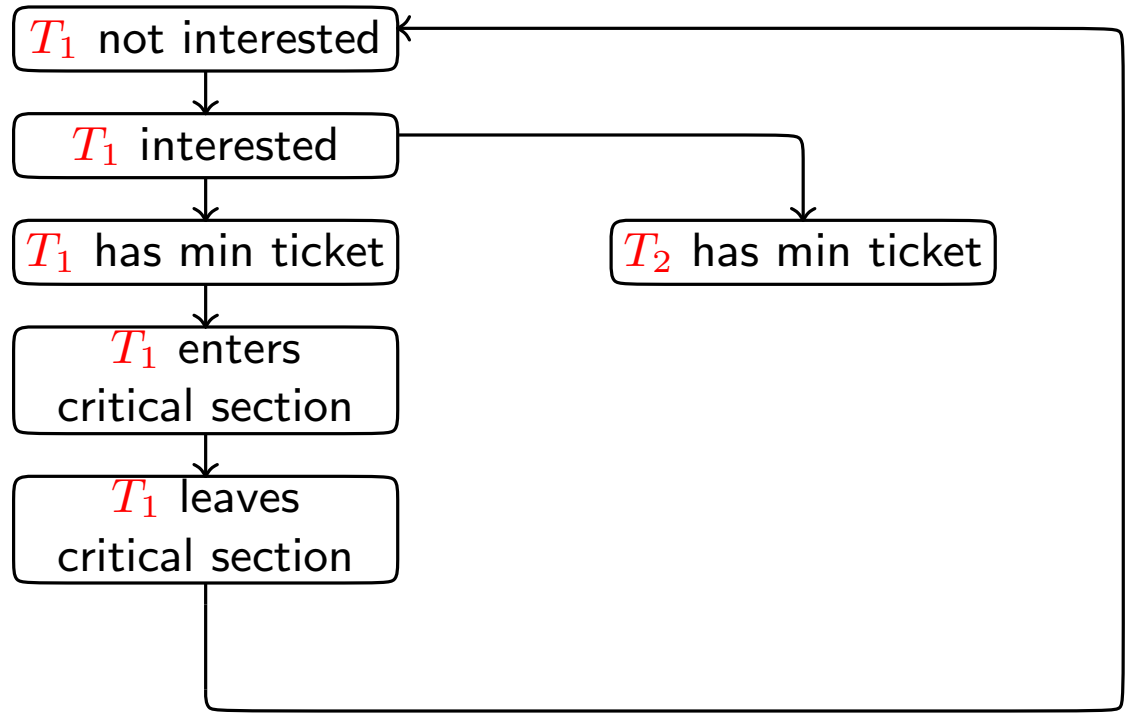

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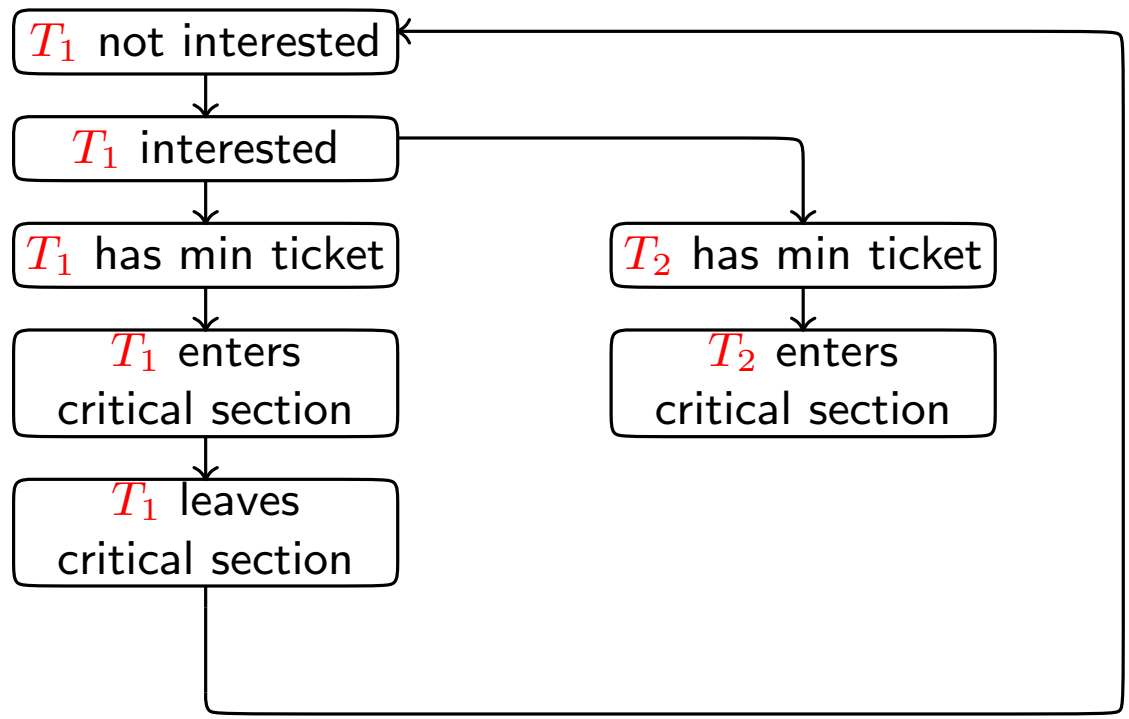

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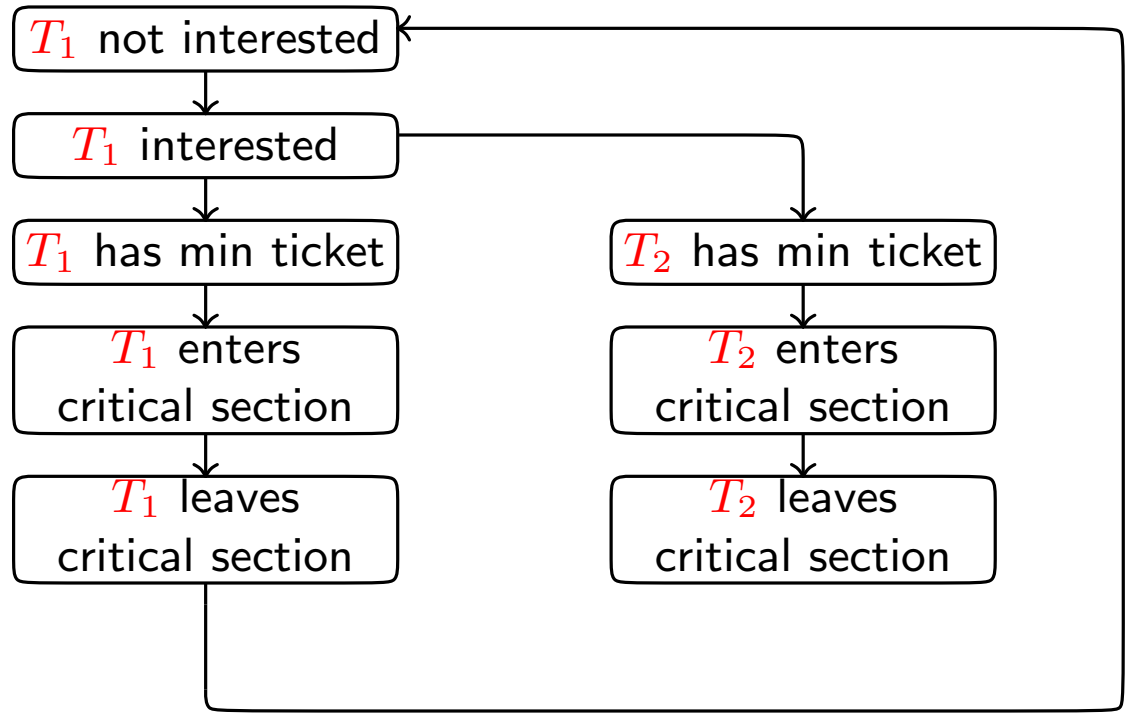

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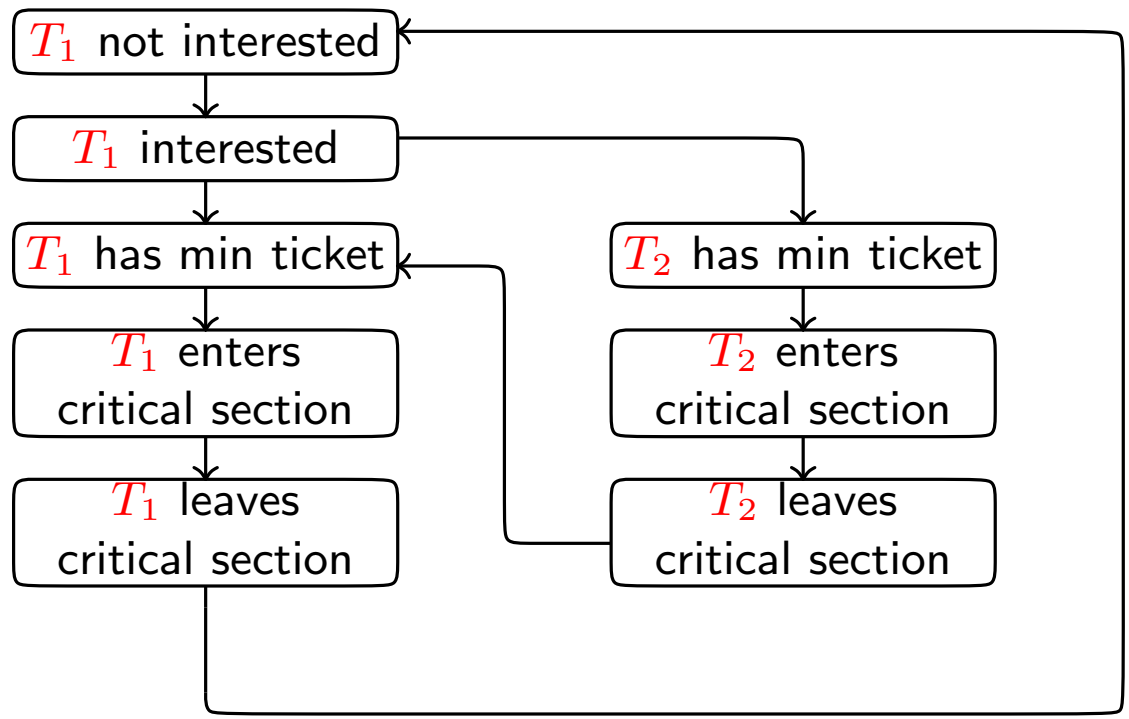

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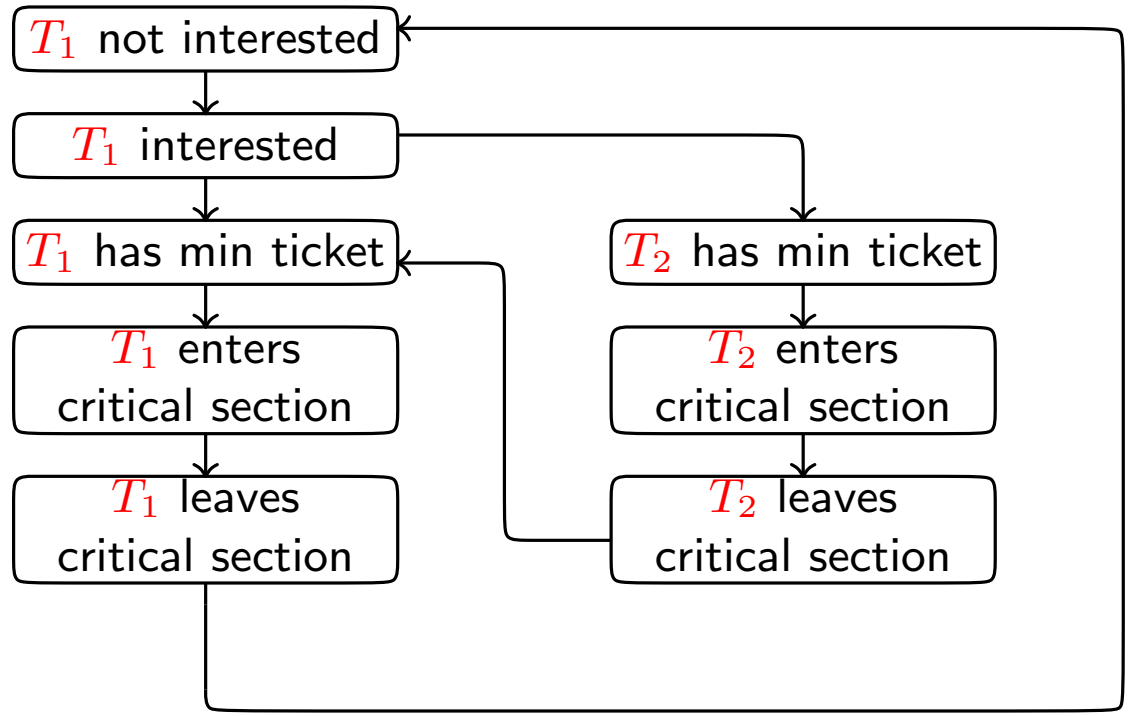

The Need of Parametrized Diagrams

System: $\textsc{SetMutex}(T_1)||\textsc{SetMutex}(T_2)||\textsc{SetMutex}(T_3)$

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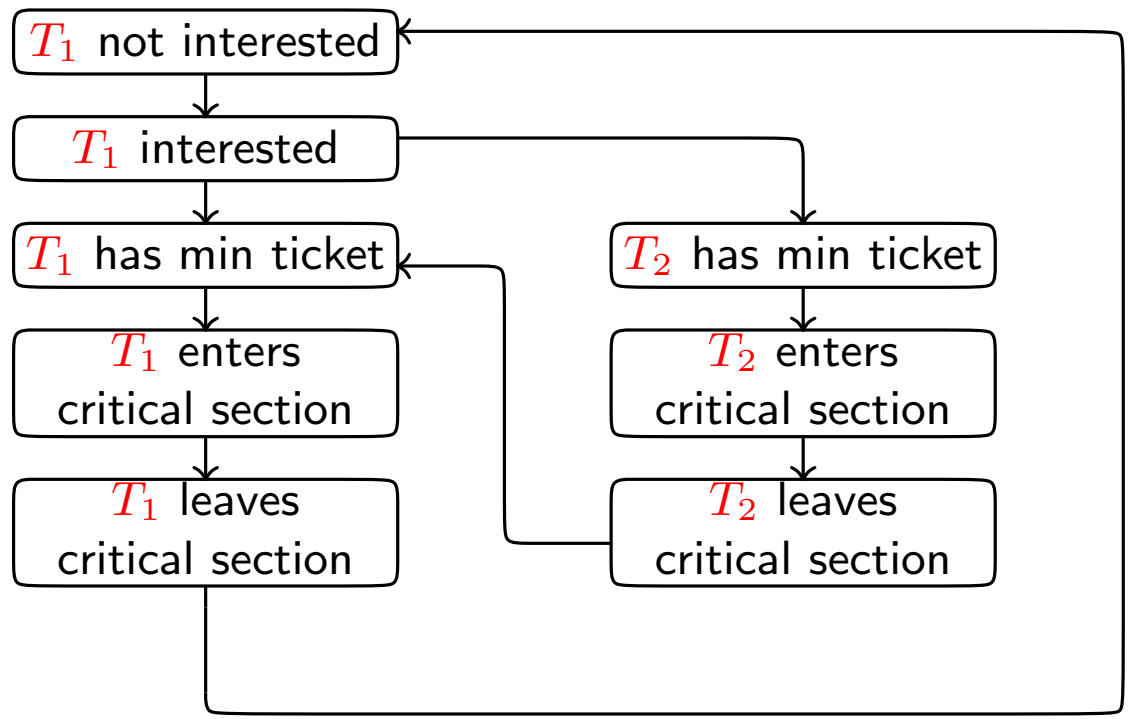

The Need of Parametrized Diagrams

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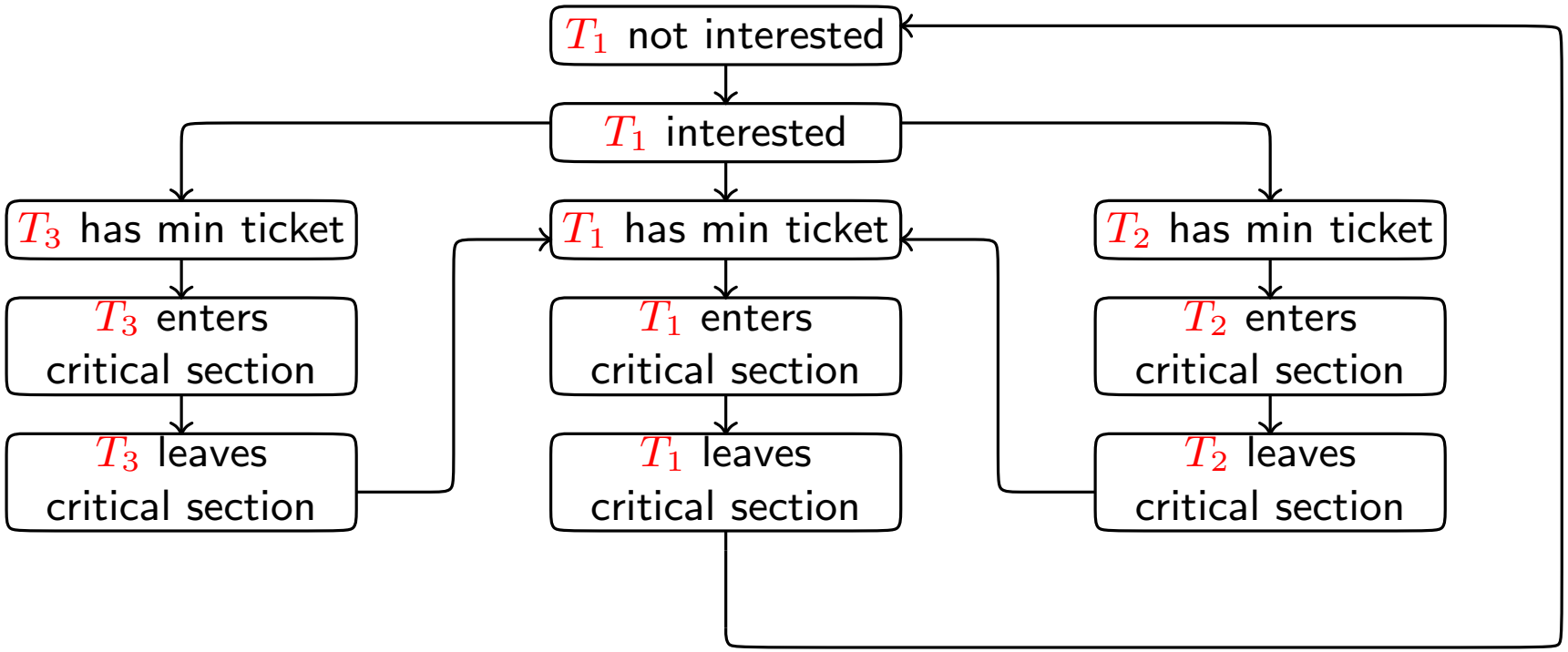

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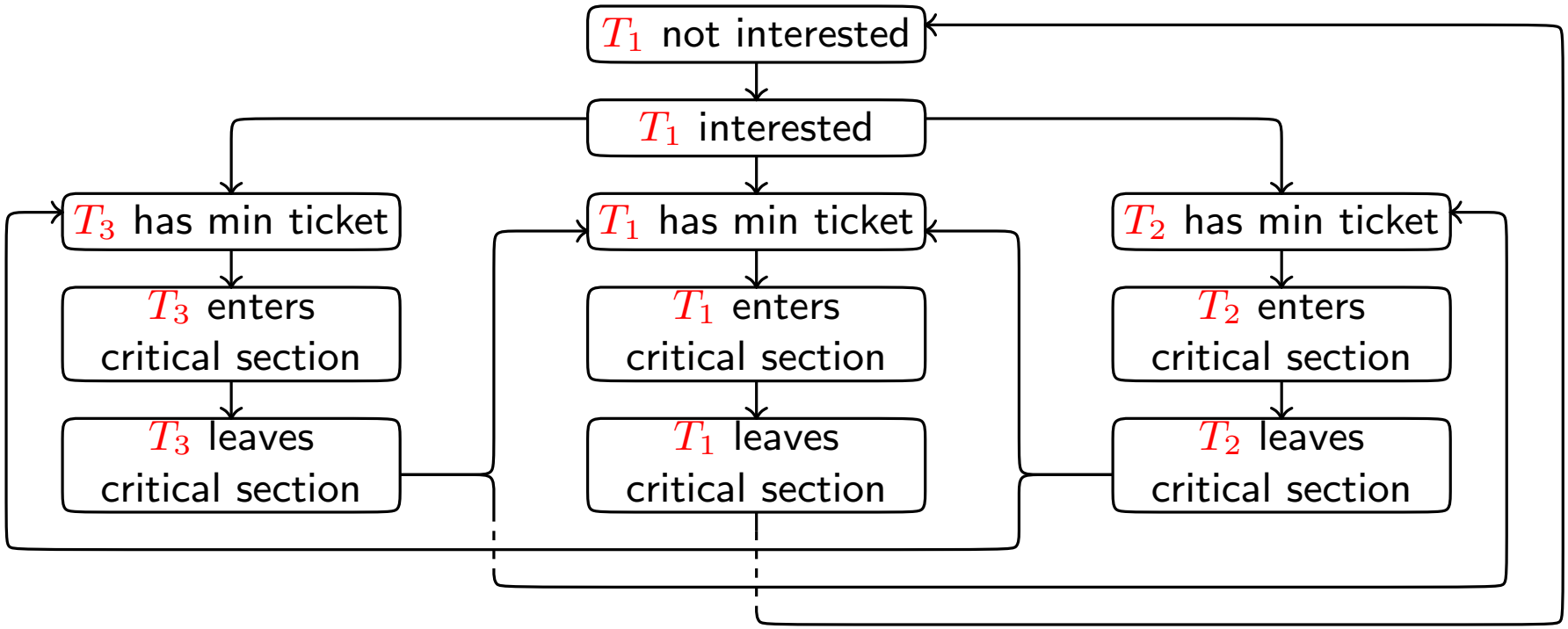

Solution: Parametrized Verification Diagrams

Problem

- **Not a single diagram** for arbitrary number of threads
- **Unbounded** number of **verification conditions**

Solution: Parametrized Verification Diagrams

Problem

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Our solution

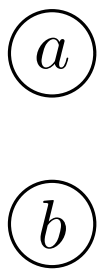
- **Unique diagram** for arbitrary number of threads
- **Finite and bounded** number of verification conditions

Parametrized Verification Diagrams exploit **symmetry**

Parametrized Verification Diagrams: Sketch

- PVDs are **an extension** of GVD

Nodes

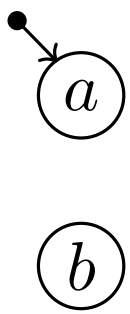


Parametrized VD

Parametrized Verification Diagrams: Sketch

- PVDs are **an extension** of GVD

Initial Node



Parametrized VD

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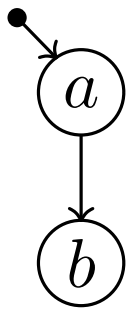
Edges



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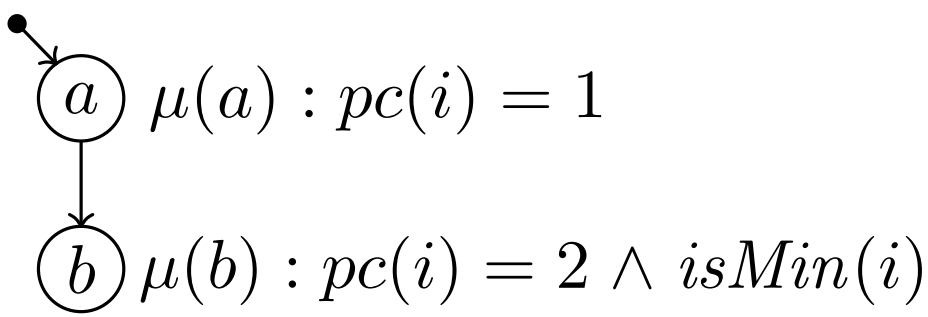
Node labeling



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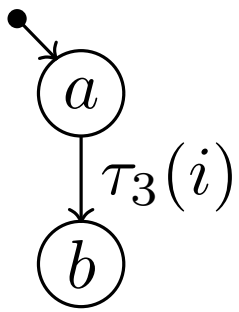

Parametrized VD

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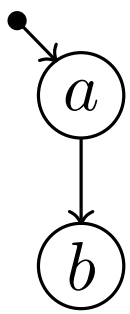
Ranking functions



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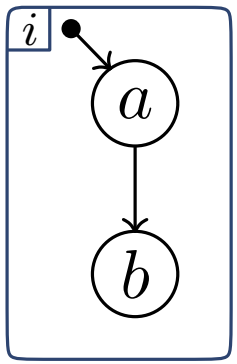

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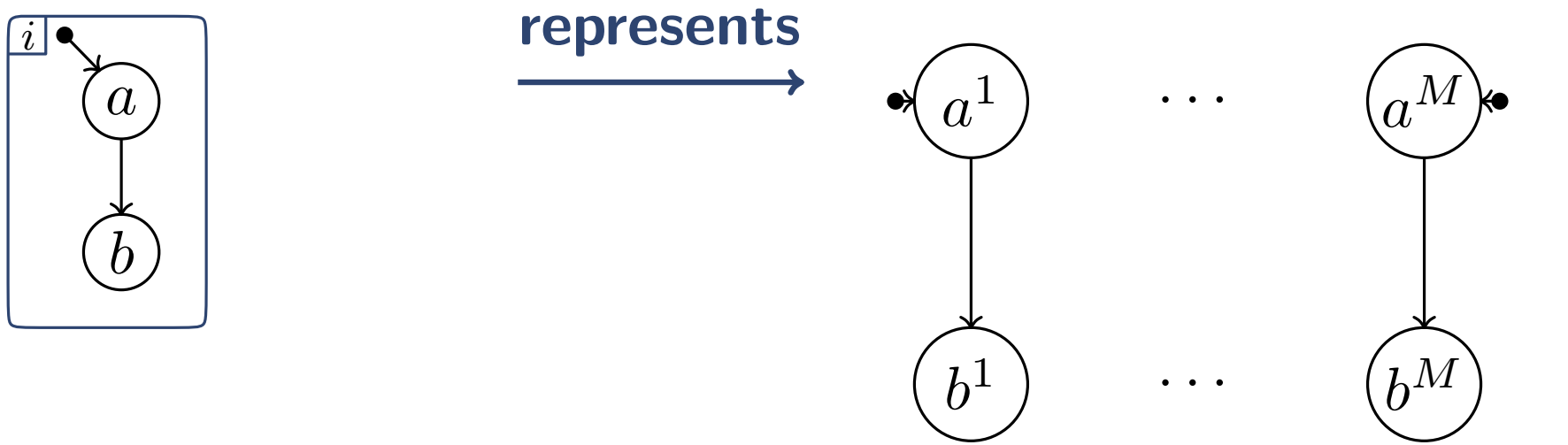
For M threads



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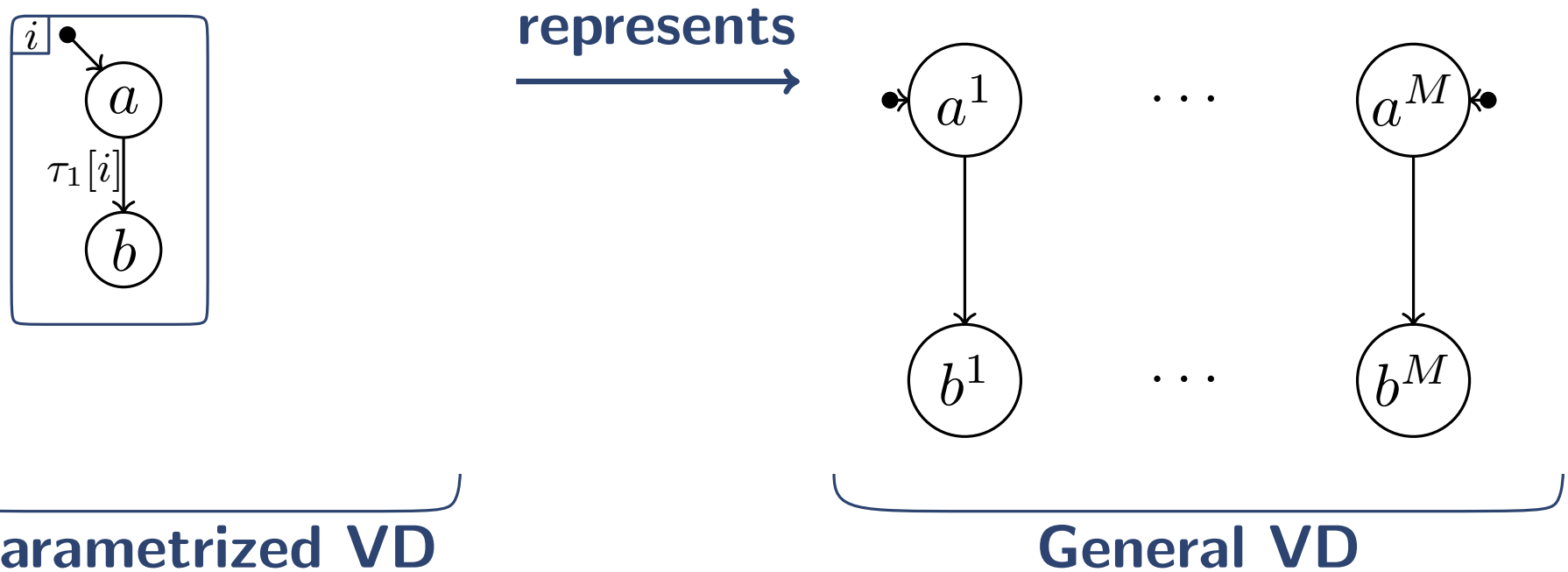

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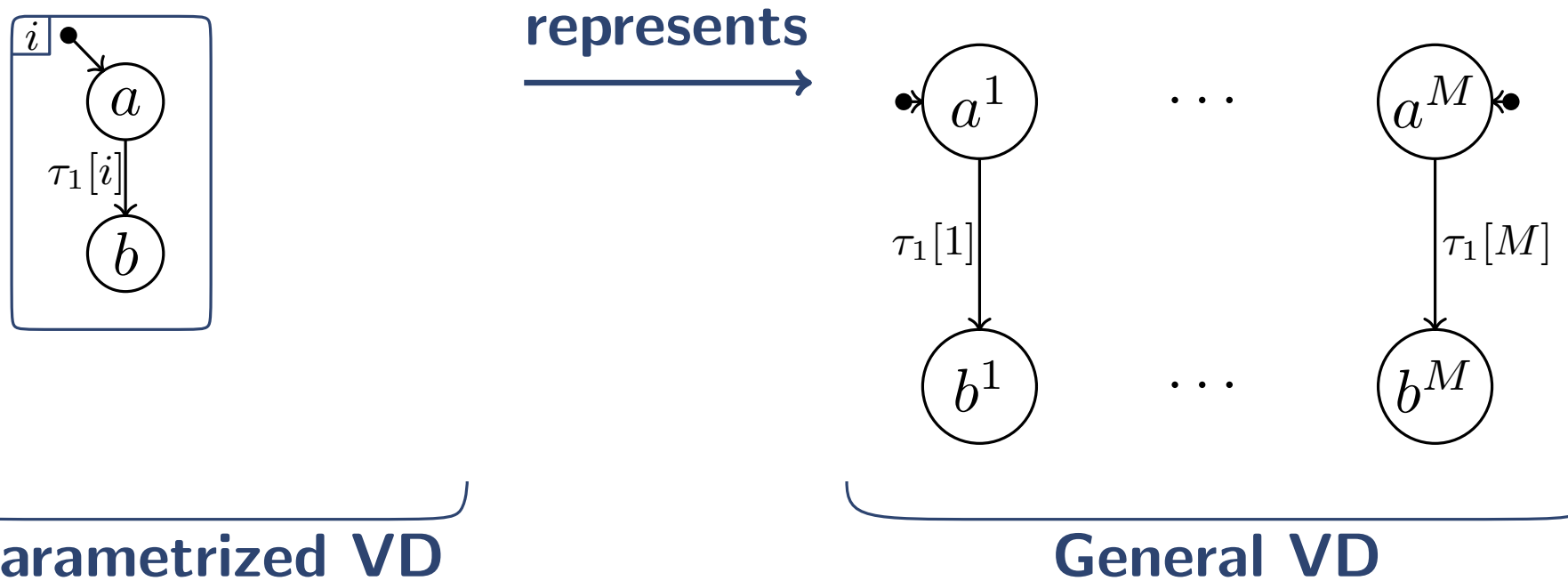

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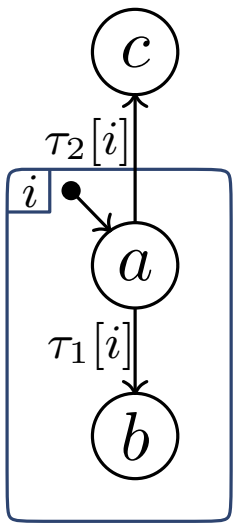
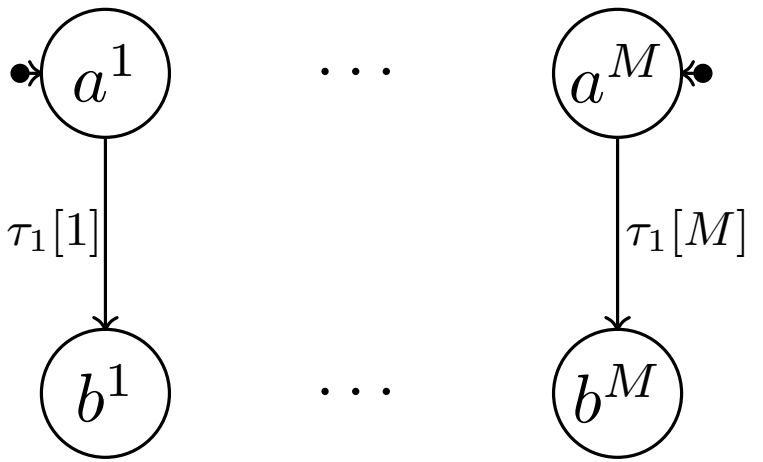


represents



Parametrized VD **General VD**

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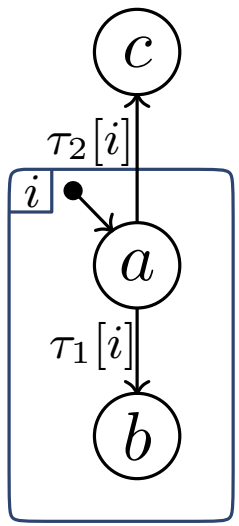

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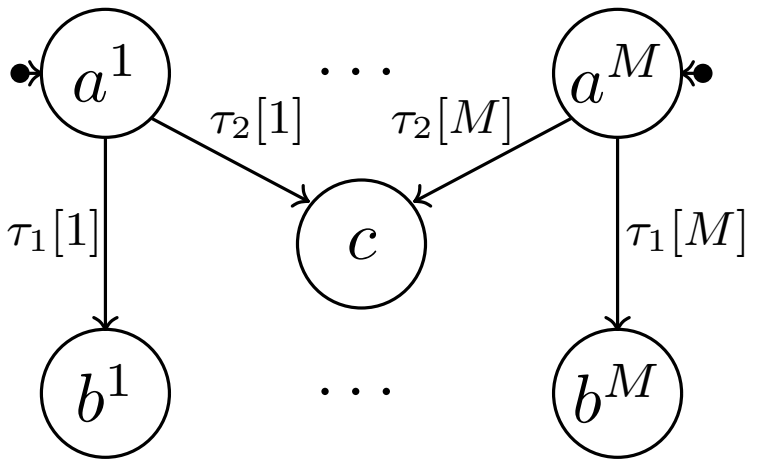

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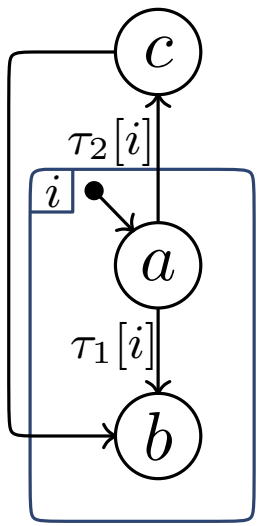

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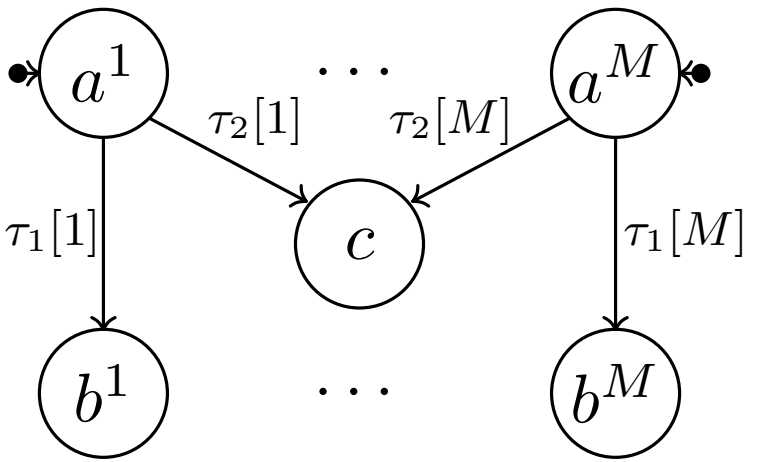

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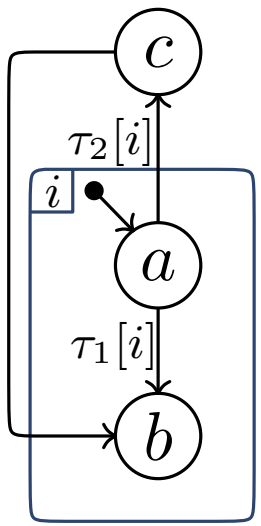

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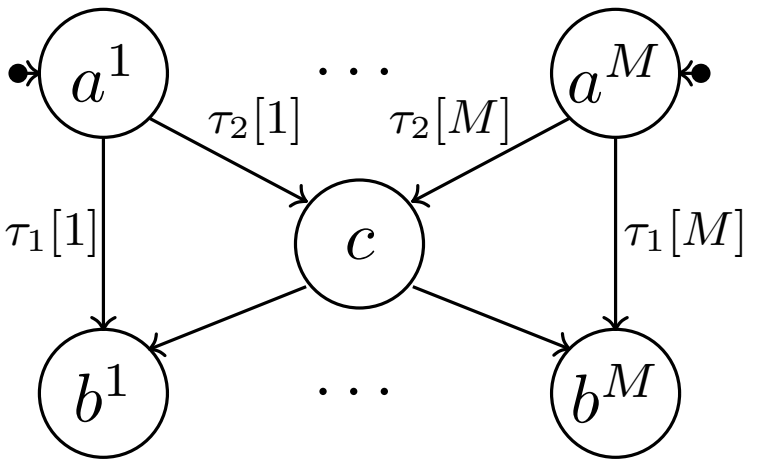

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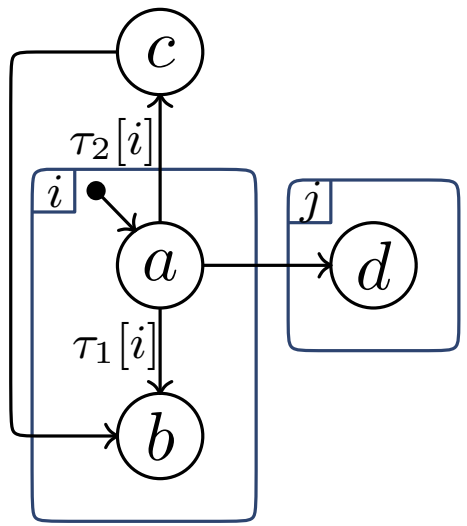

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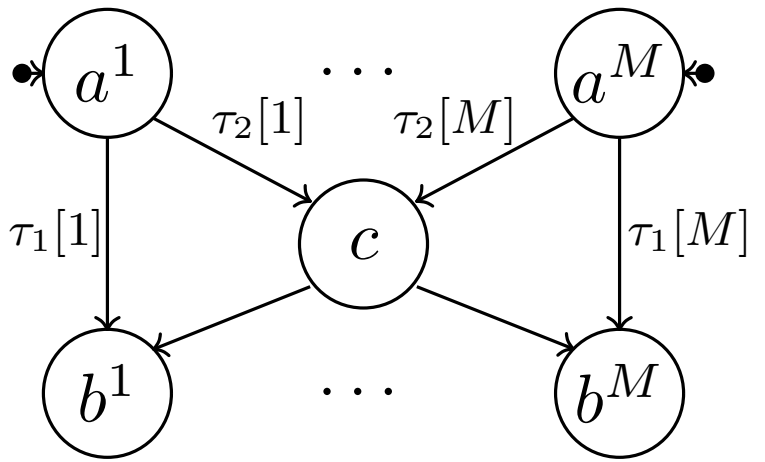

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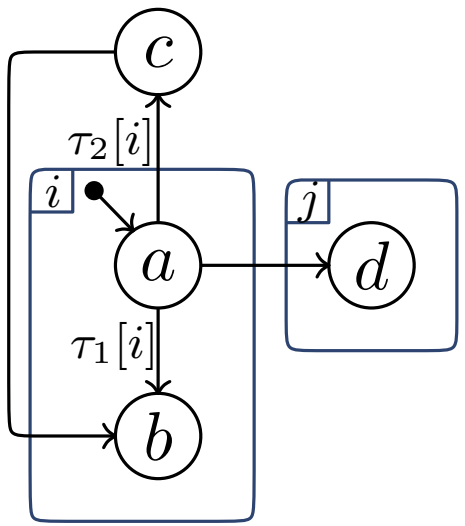

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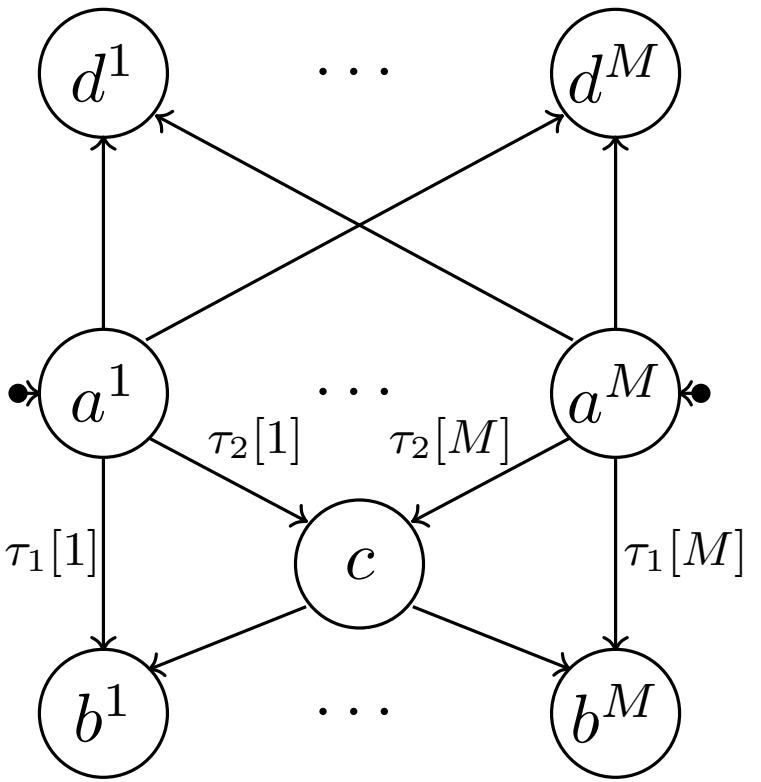

Parametrized VD **General VD**

Parametrized Verification Diagrams: Sketch

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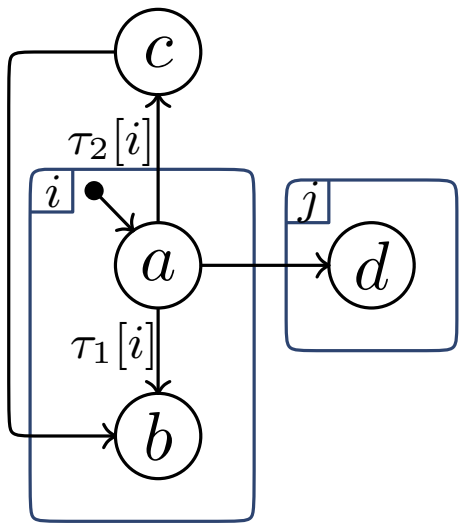
A PVD **abstracts all instantiations** of a parametric system

For M threads



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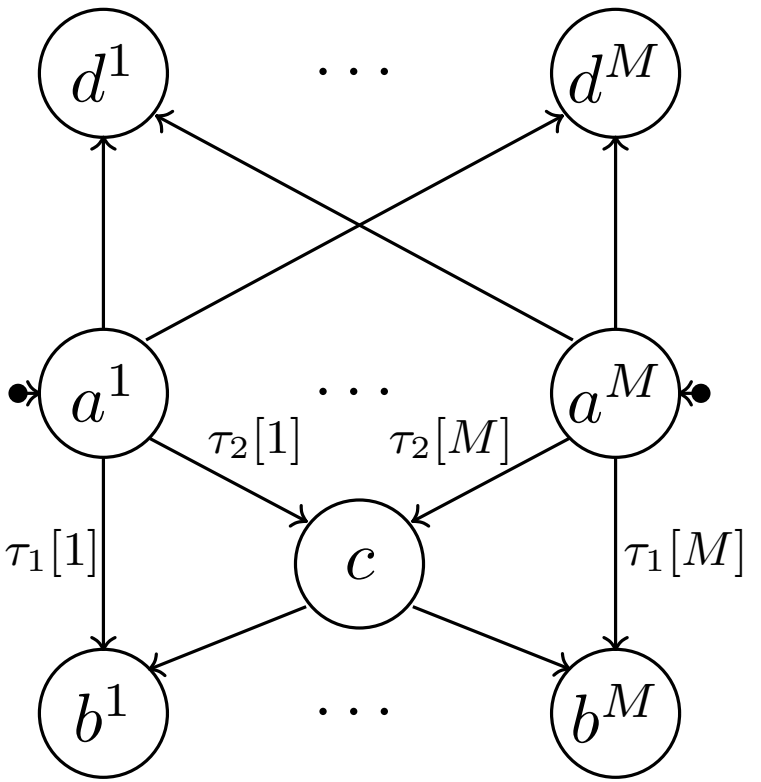
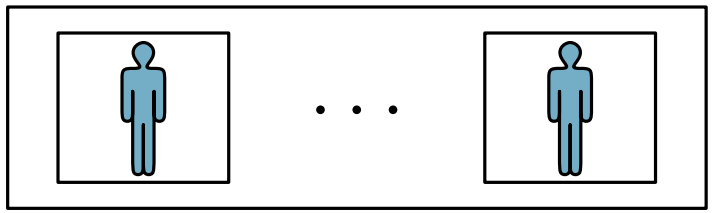


Parametrized VD **General VD**

Verification Conditions for Parametrized Diagrams



$P[N] : P(1)|| \cdots ||P(N)$

Diagram Property

$\models \quad \mathcal{D} \quad \models \quad \varphi^{(k)}$

Verification Conditions:

- Initiation
- Consecution
- Acceptance
- Fairness

Verification Conditions for Parametrized Diagrams

- **Initiation** $\Theta \rightarrow \mu(N_0)$

Verification Conditions for Parametrized Diagrams

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- **Consecution:** For every $n \in N$ and $\tau \in \mathcal{T}$,

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How many VCs?

$|\mathcal{T}| \cdot |\mathcal{V}|$

Self-Consecution

$$\bigvee_{n \to m} \mu(n) \wedge \tau(i) \;\rightarrow\; \mu(m')$$

for all $i \in Voc(n, m)$

$|\mathcal{T}|$

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$$(\mu(n) \wedge \tau(i) \wedge \mu(m')) \to \delta(n) > \delta(m) \qquad \text{if } e \in B$$

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$$\mu(n) \rightarrow En(\tau(i))$$

$$\mu(n) \wedge \tau(i) \rightarrow \mu(m')$$

Verification Conditions for Parametrized Diagrams

- **Initiation**
- **Consecution**
- **Acceptance**
- **Fairness**
- **Satisfaction**

$$\mu(n) \to f(n)$$

$$\mathsf{ModelCheck}(\mathcal{D} \models \varphi)$$

Mutual Exclusion Algorithm (revisited)

$$\varphi(k) \;=\; \Box(pc(k) = 3 \rightarrow \Diamond pc(k) = 5)$$

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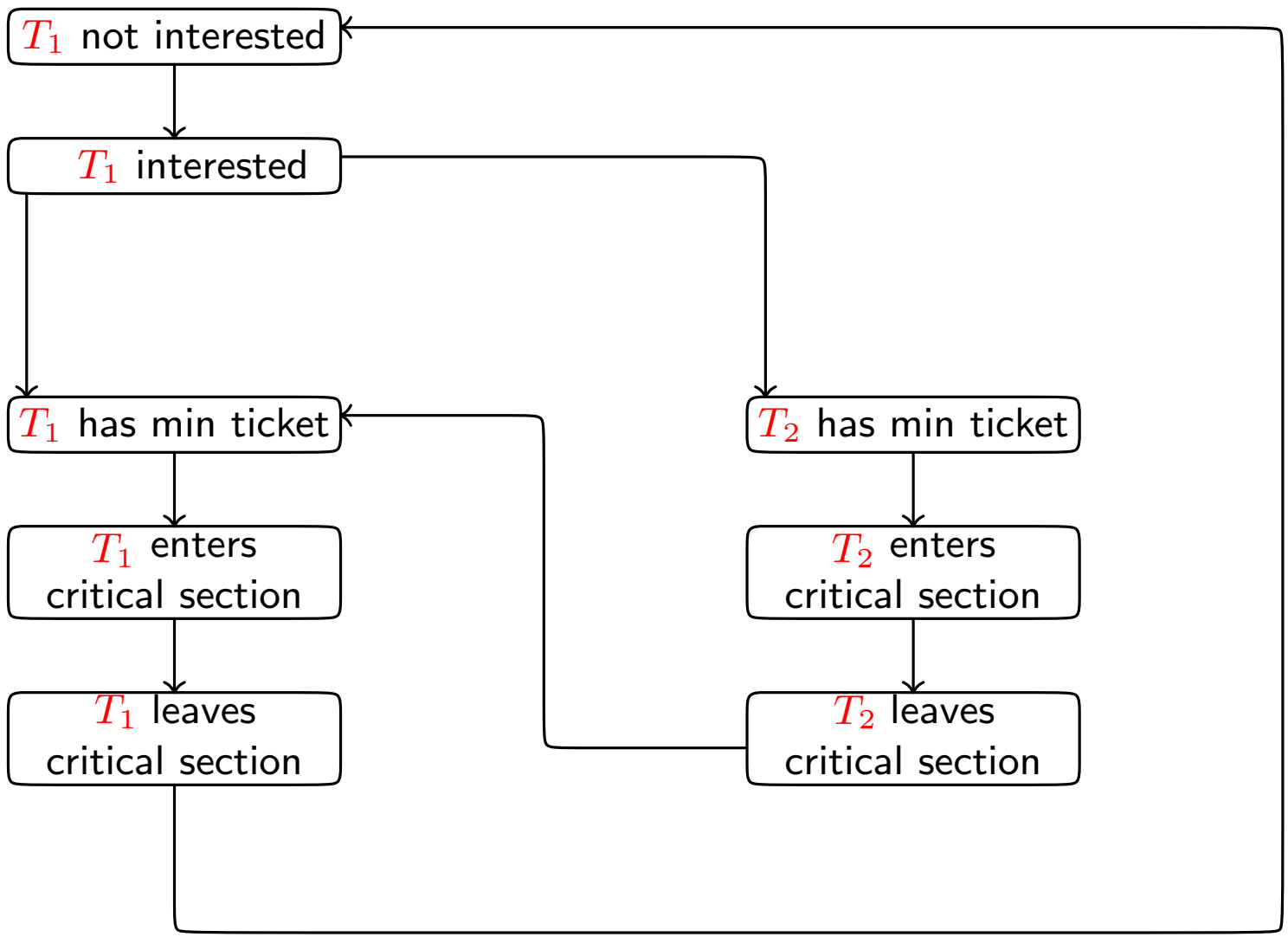

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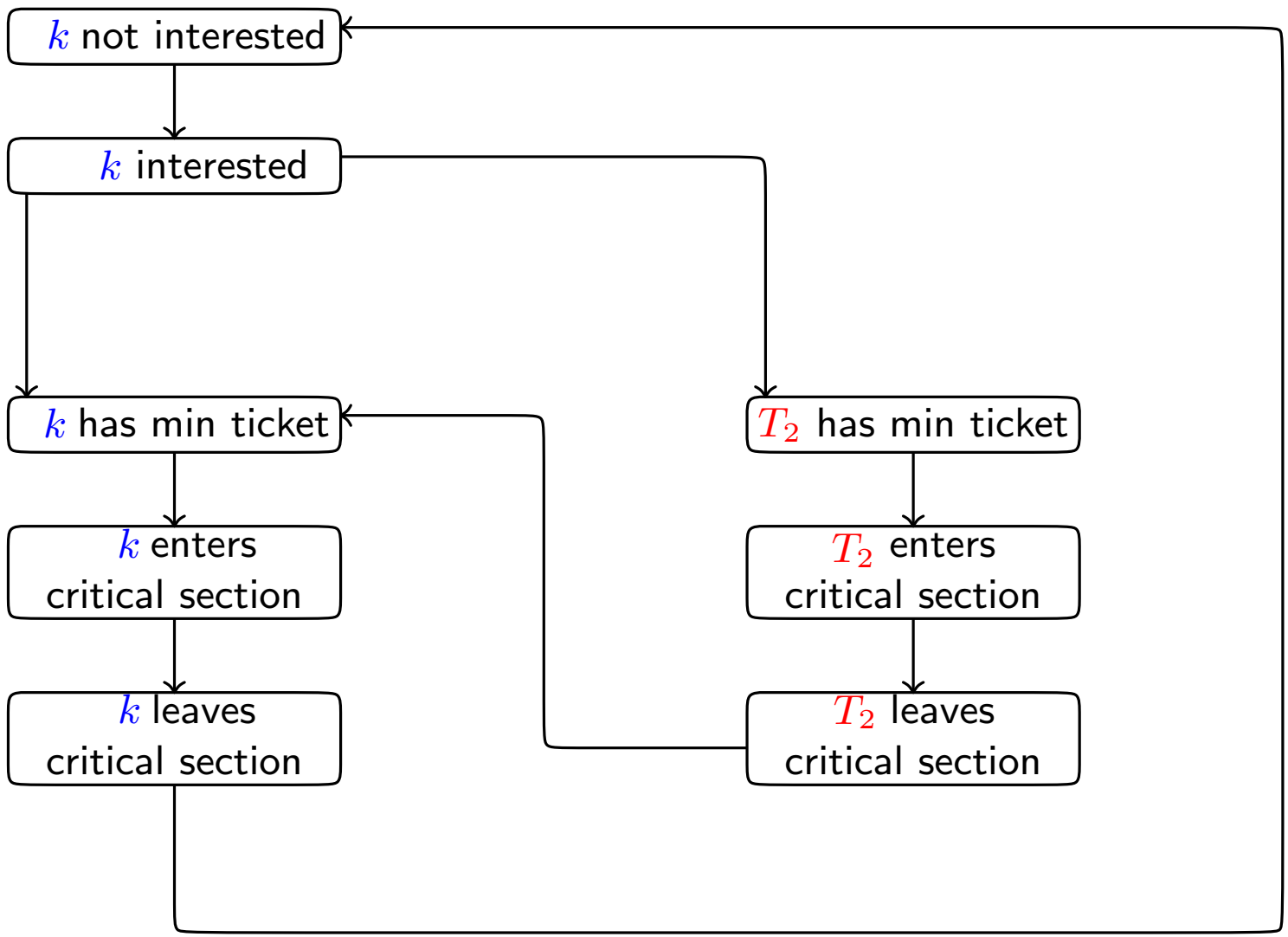

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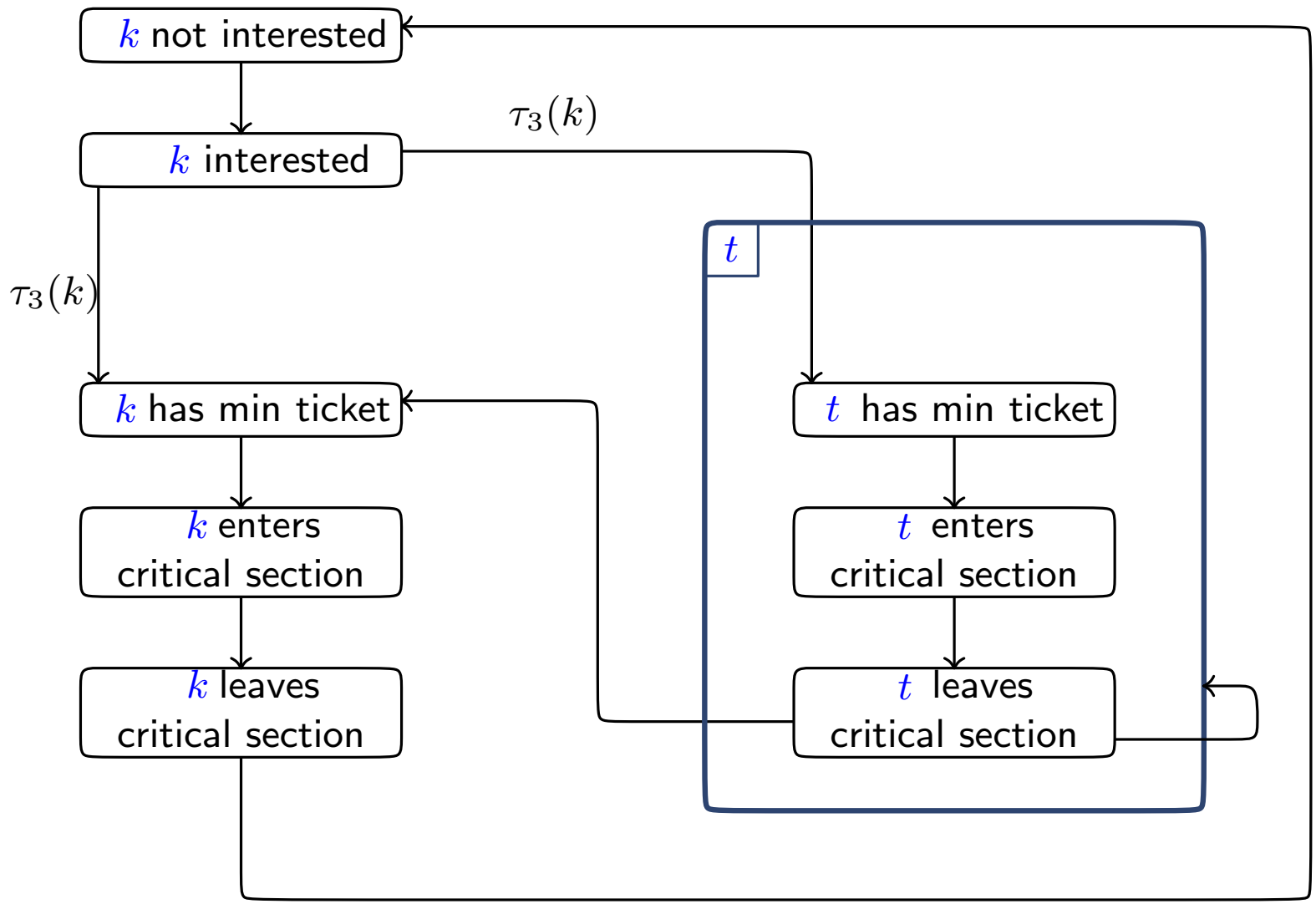

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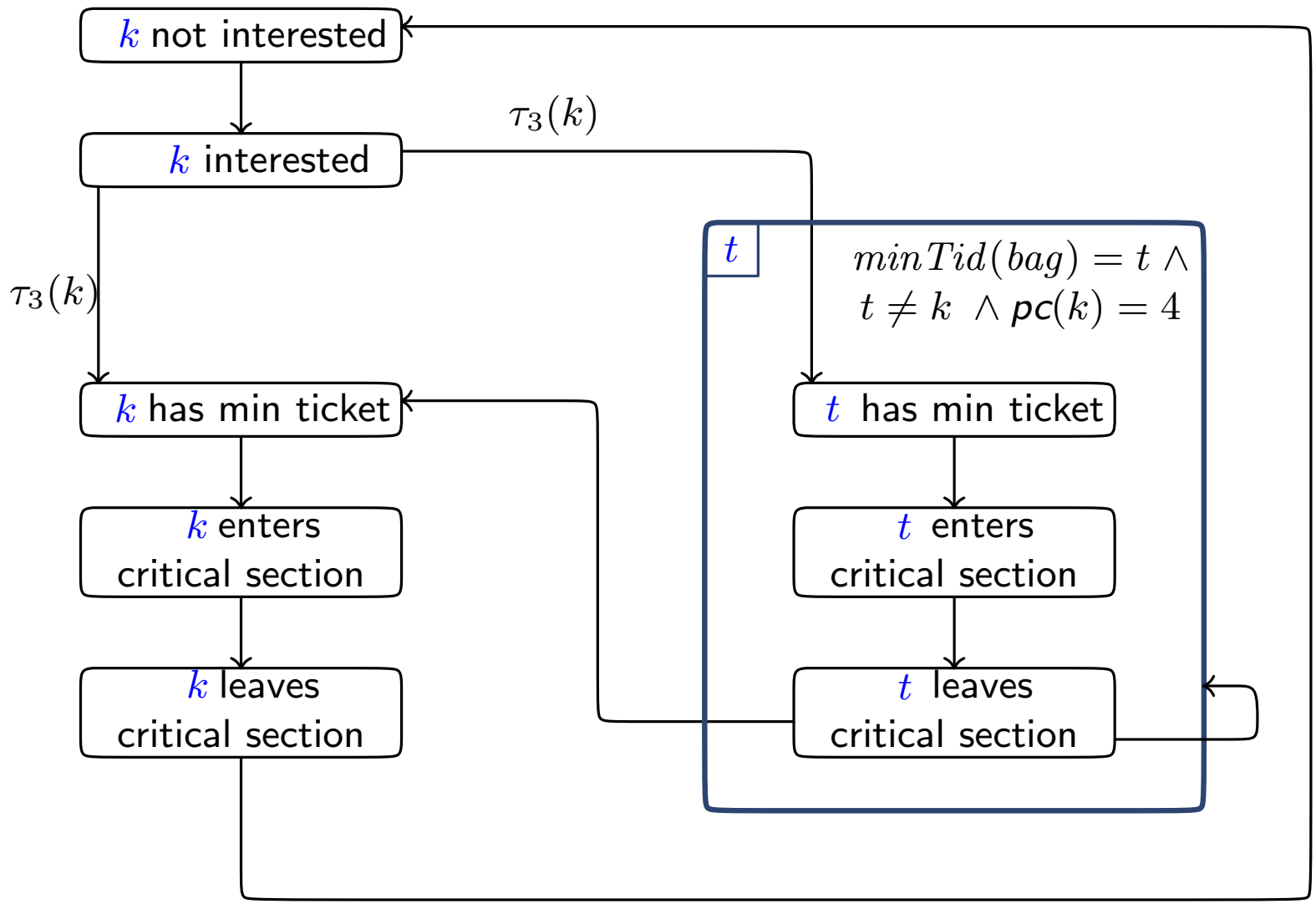

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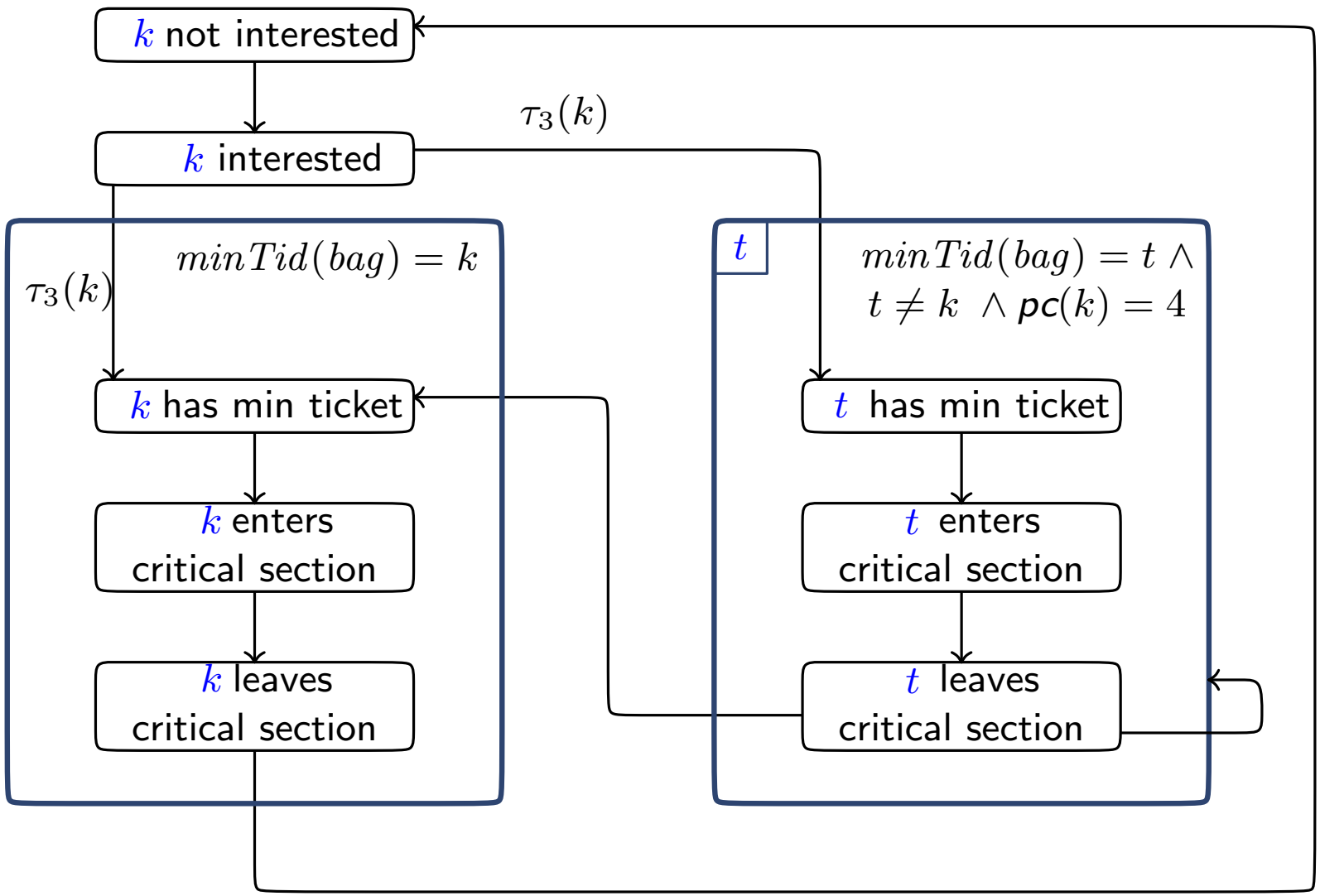

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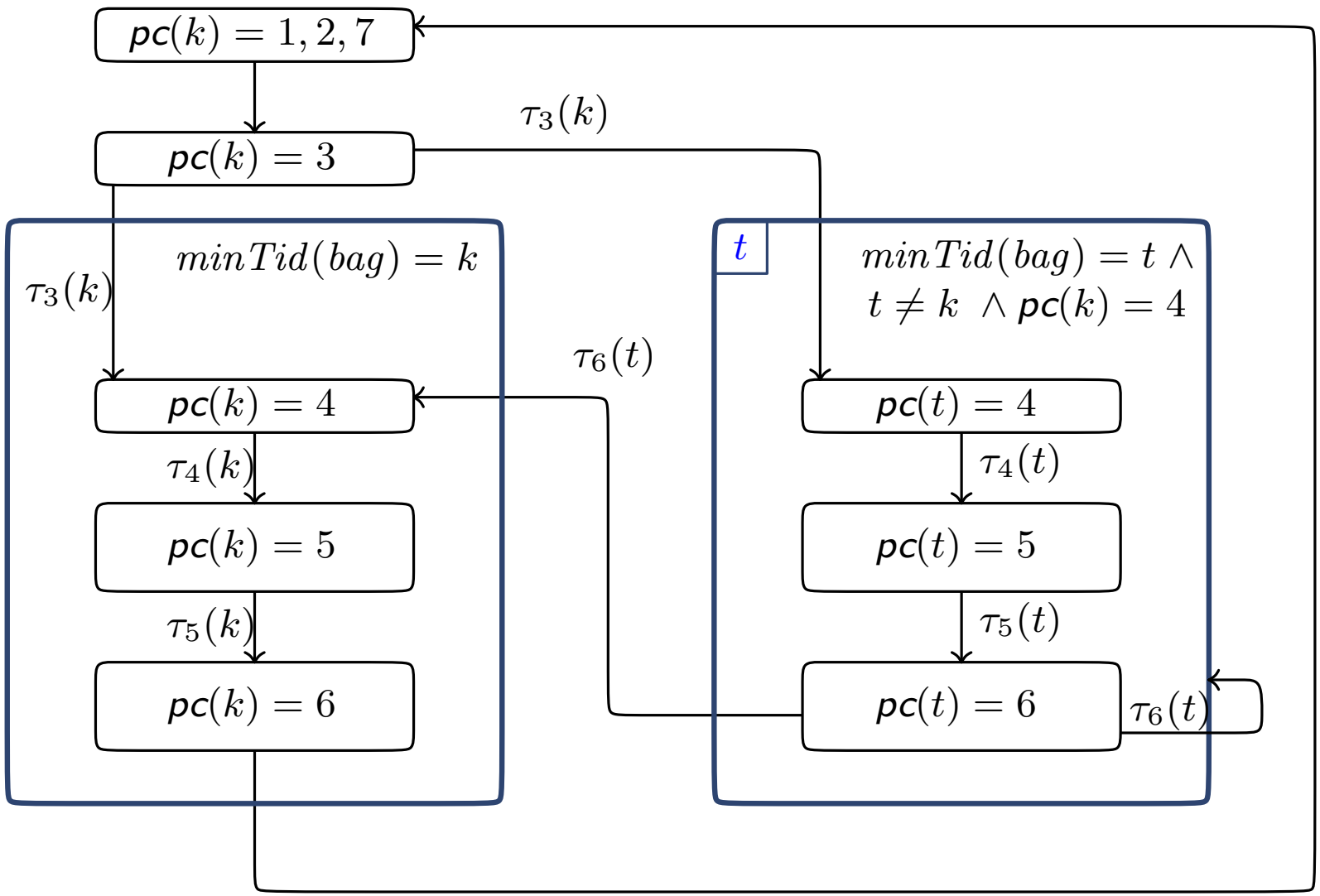

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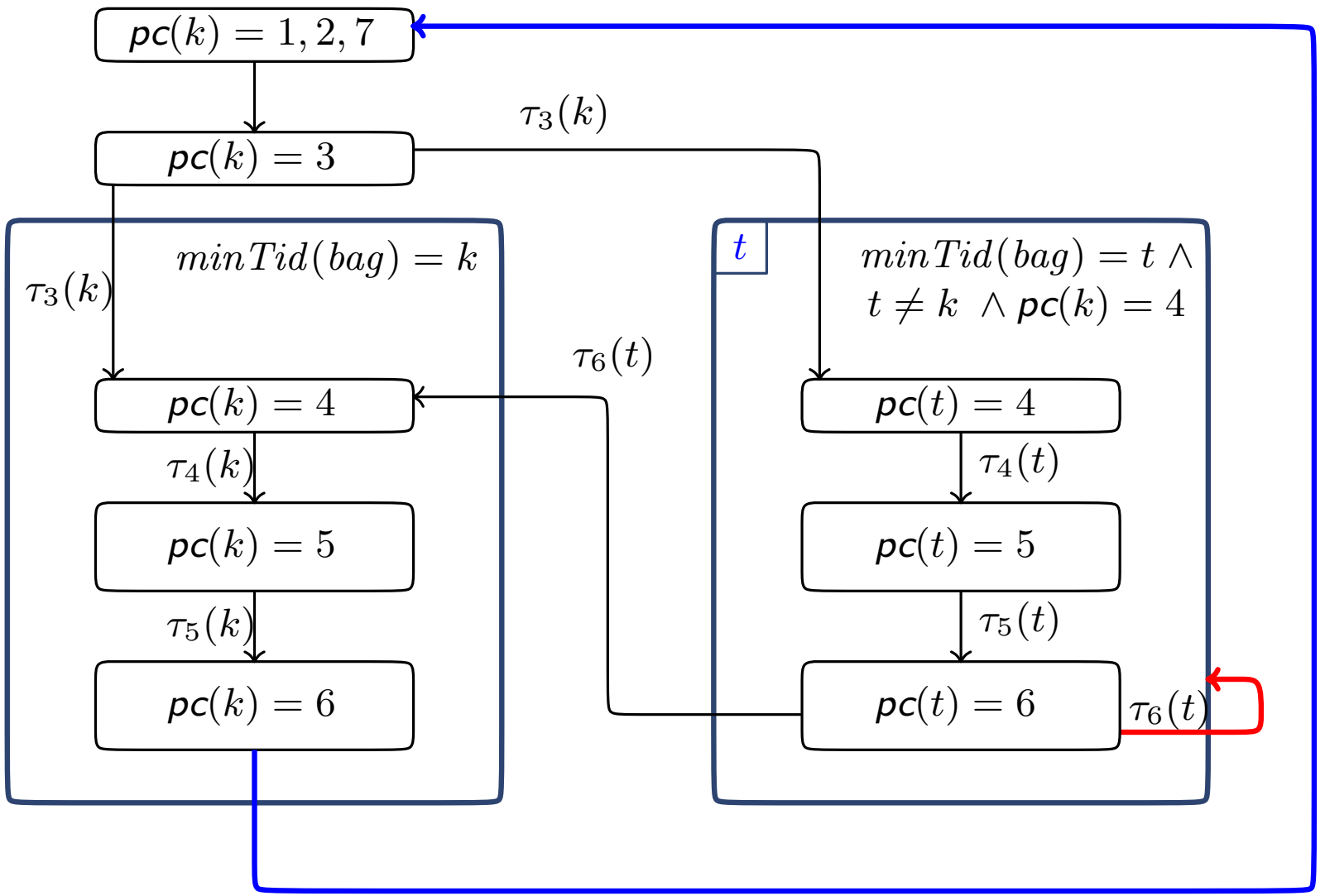
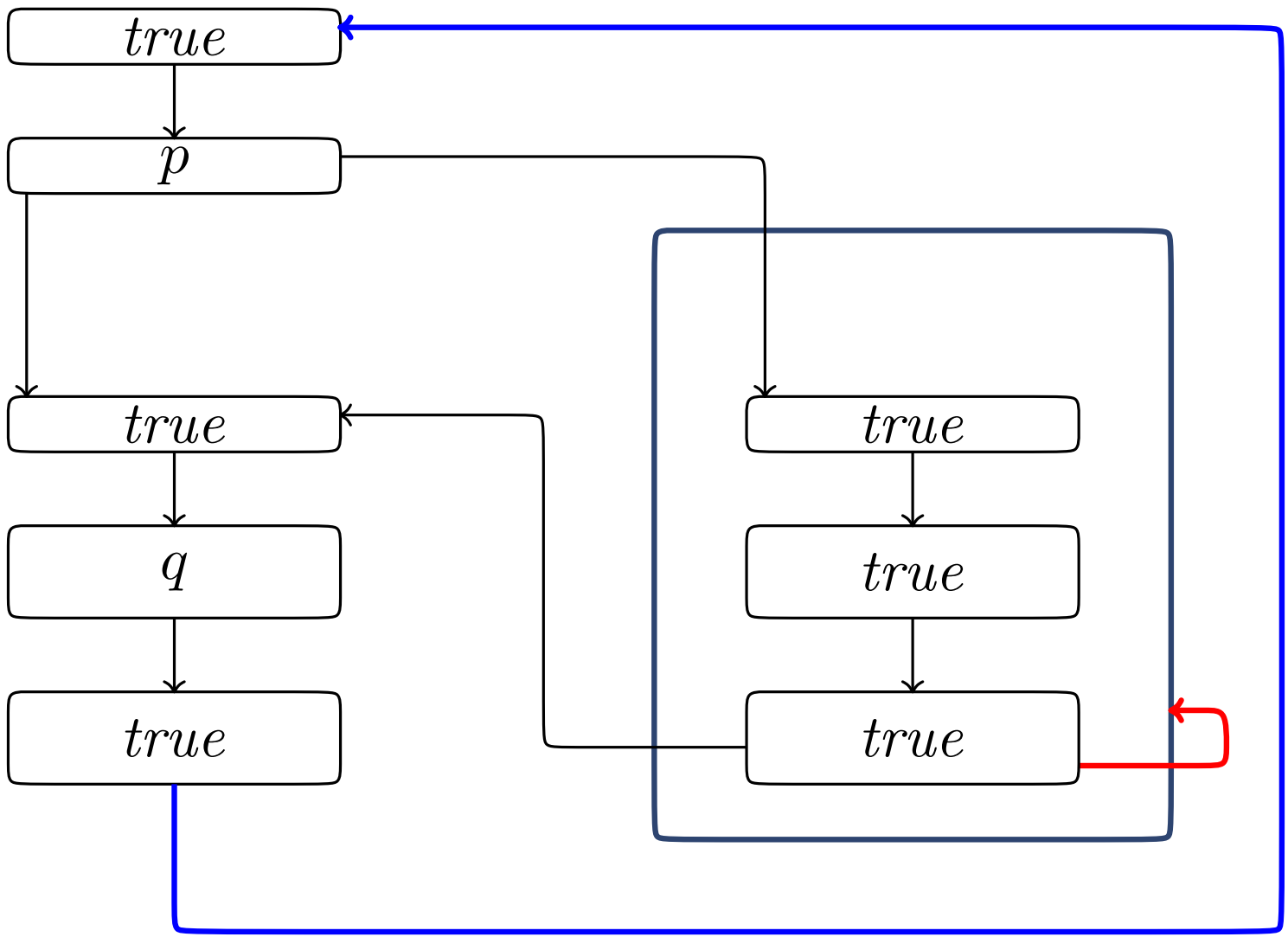
B, G $\quad \delta : lower(bag, ticket(k))$



Mutual Exclusion Algorithm (revisited)

$$\varphi(k) = \Box(\quad p \quad \rightarrow \Diamond \quad q \quad)$$



Our Contributions

A Deductive Verification Techniques for Parametrized Systems

1 Parametrized Invariance
Deductive proof rules for concurrent parametrized invariants

2 Parametrized Verification Diagrams
Diagram based verification for concurrent parametrized liveness properties

3 Invariant Generation using Self-reflection
Automatic parametrized invariant generation using off-the-shelf sequential absint

B Decision Procedures for Complex Data Structures

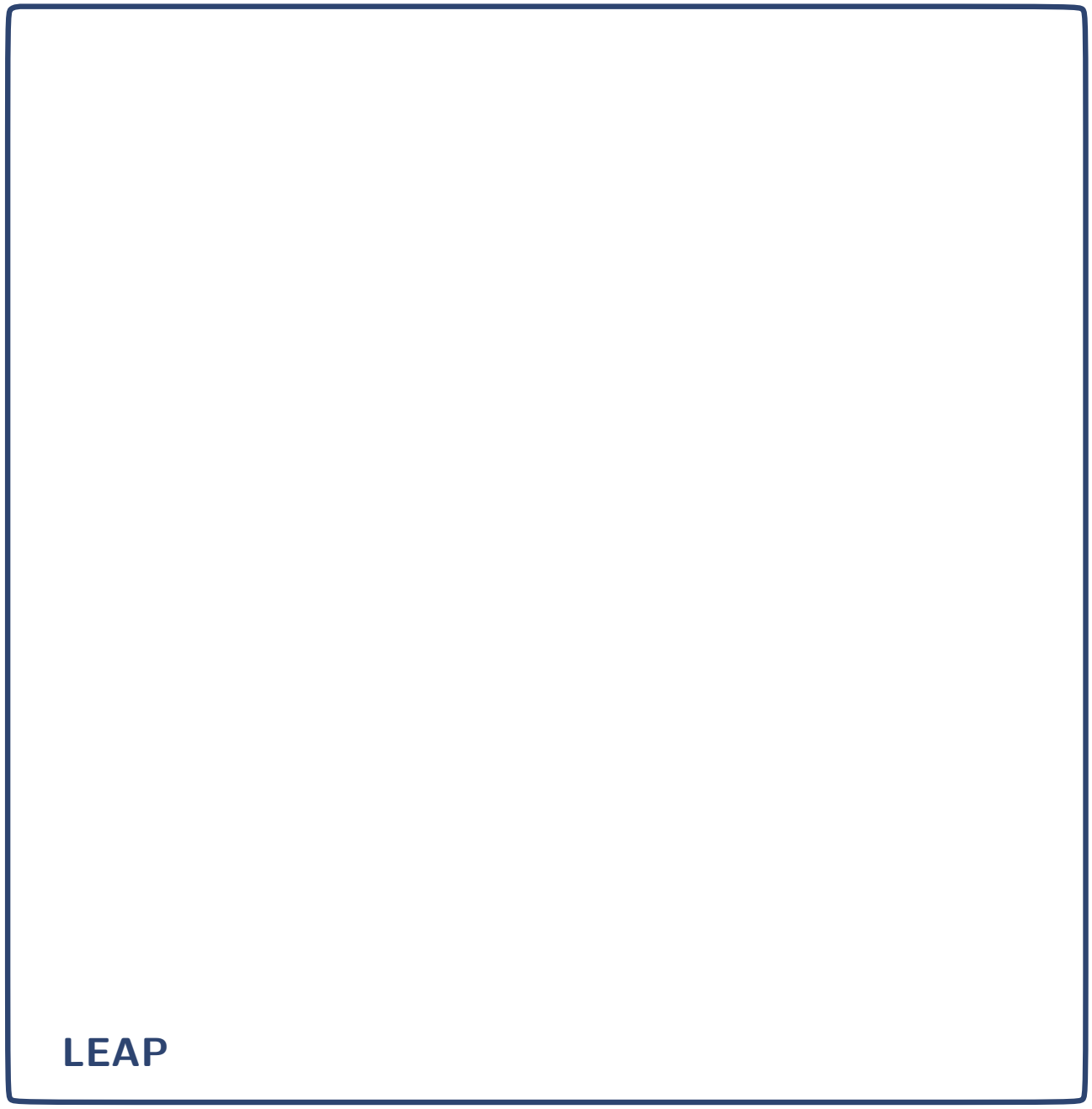
4 TL3: A Decidable Theory for Concurrent Lists
A Theory and decision procedure for concurrent data structures of the shape of a list

5 TSL_K: A Decidable Family for Concurrent Bounded Skiplists
Theories and decision procedures for concurrent skiplists of at most K levels

6 TSL: A Decidable Theory for Skiplists with Arbitrary Levels
Theory and decision procedure for skiplists with unbounded many levels

C Implementation and Evaluation of our Framework

LEAP: Structure

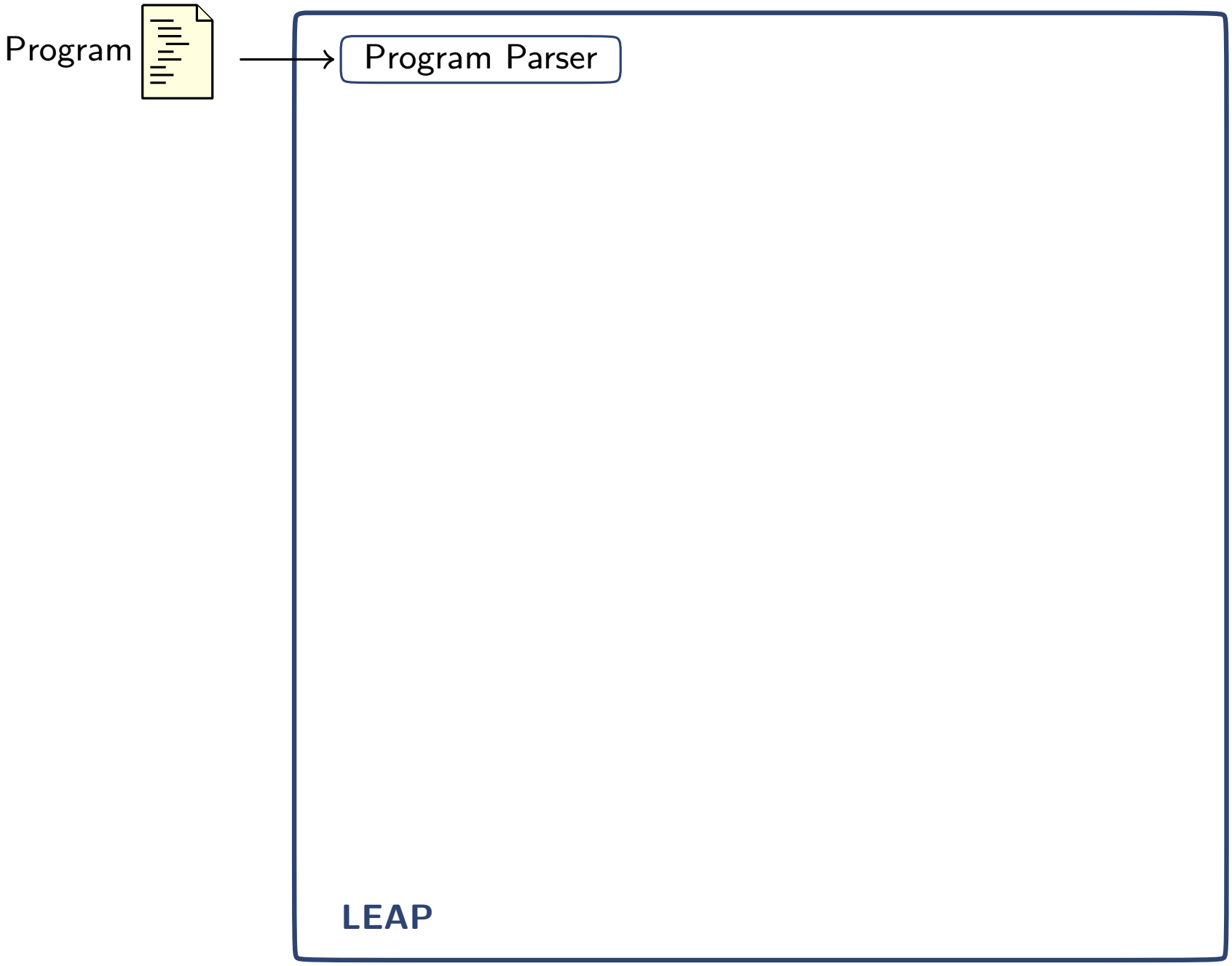


LEAP: Structure

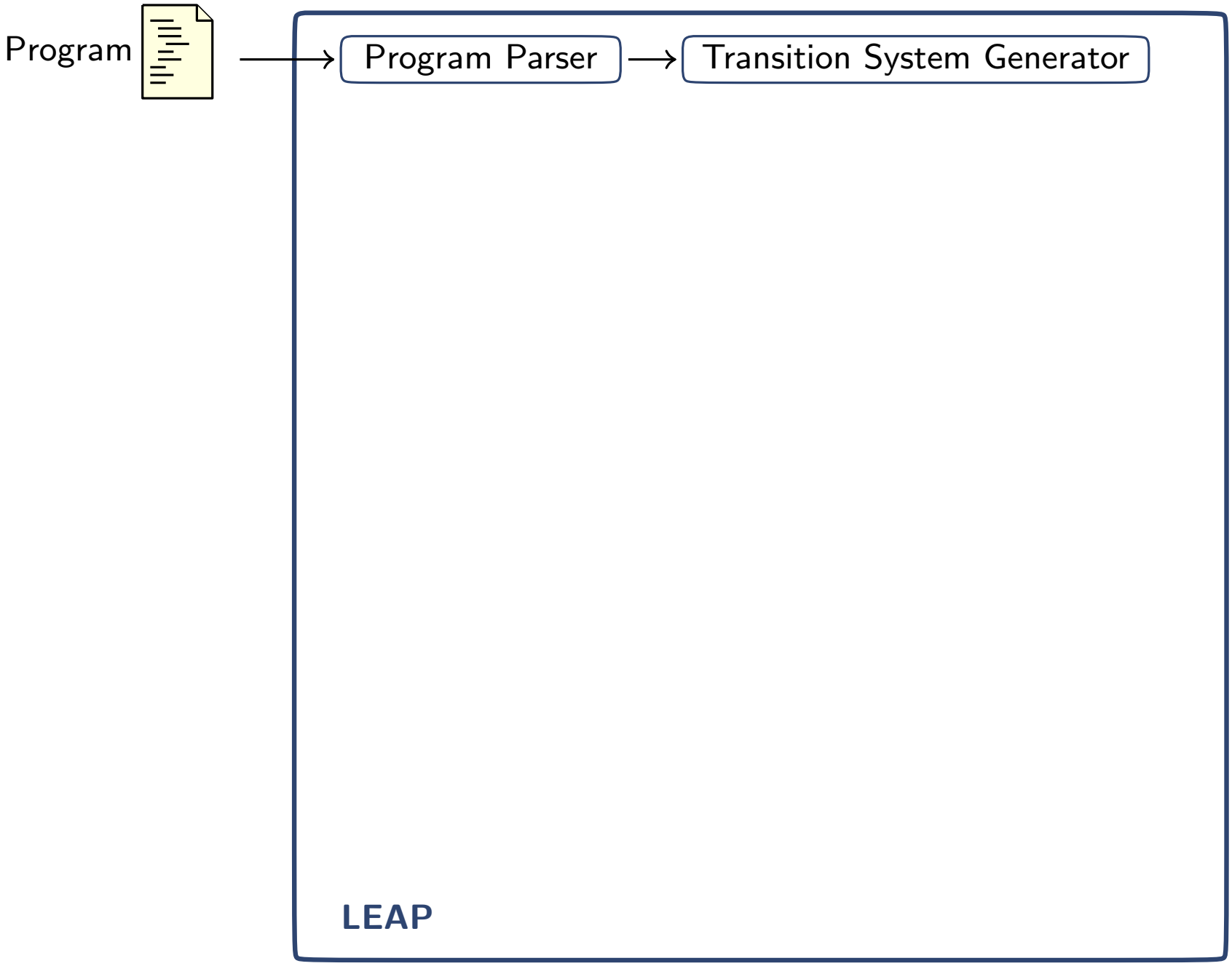
Program 

LEAP

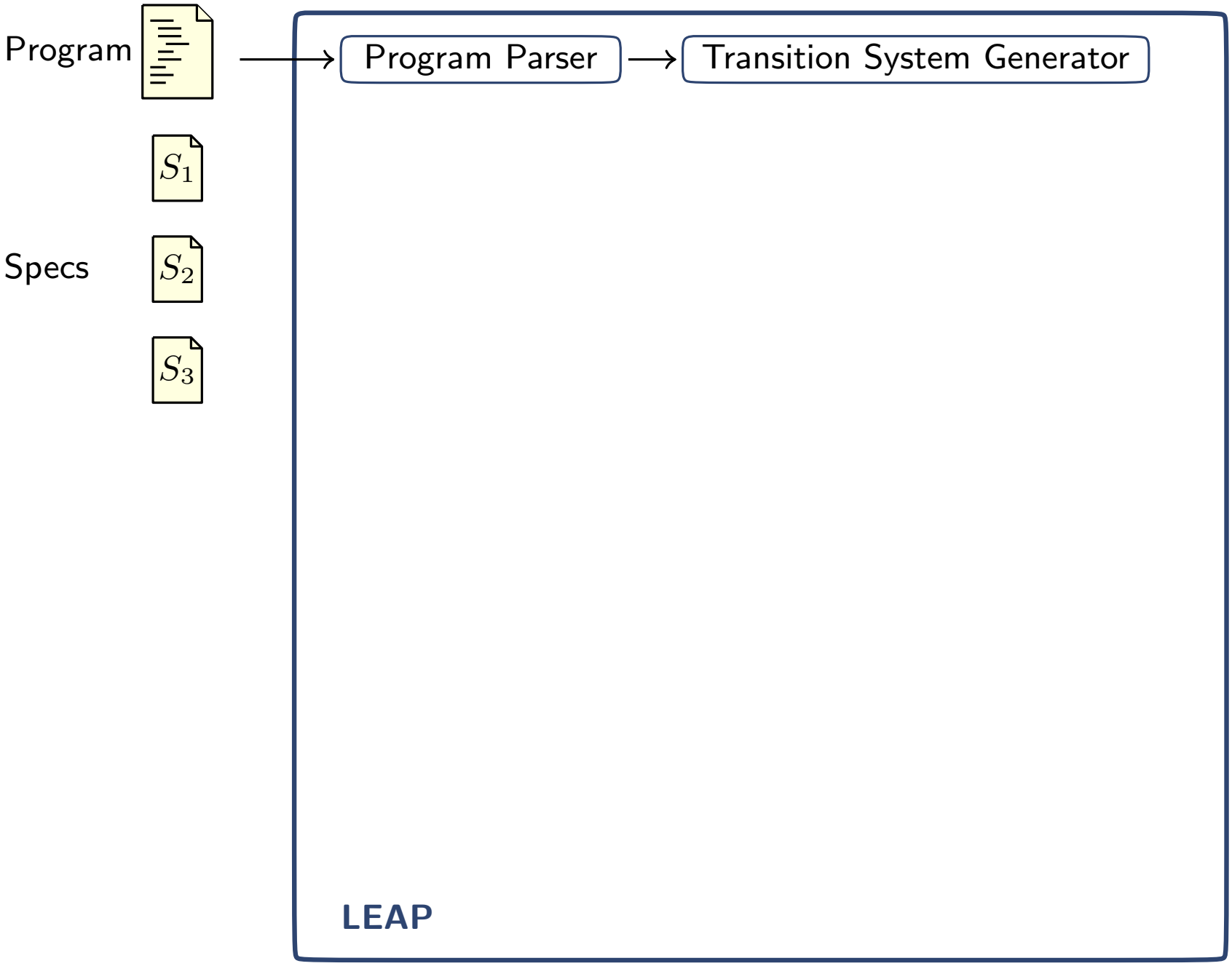
LEAP: Structure



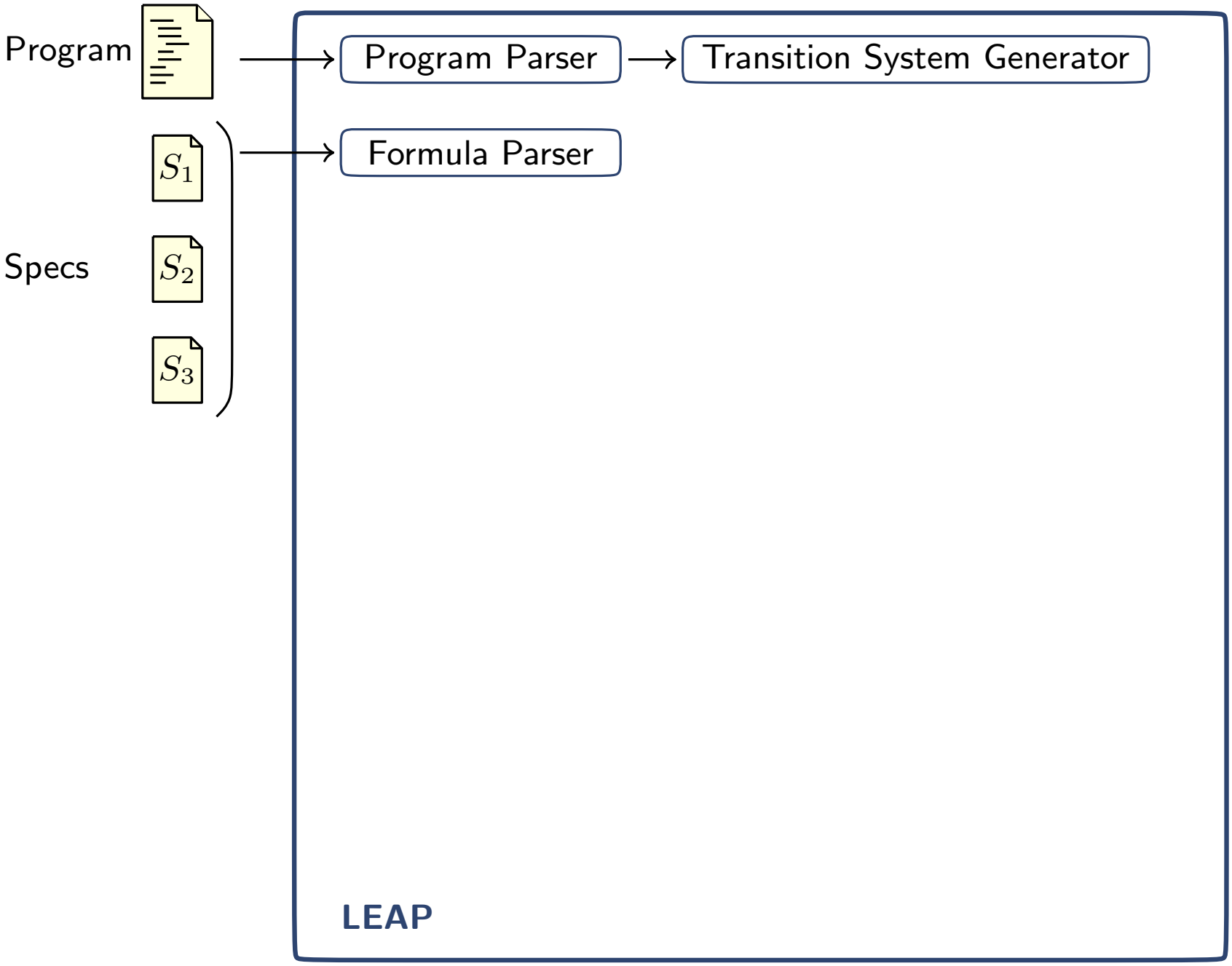
LEAP: Structure



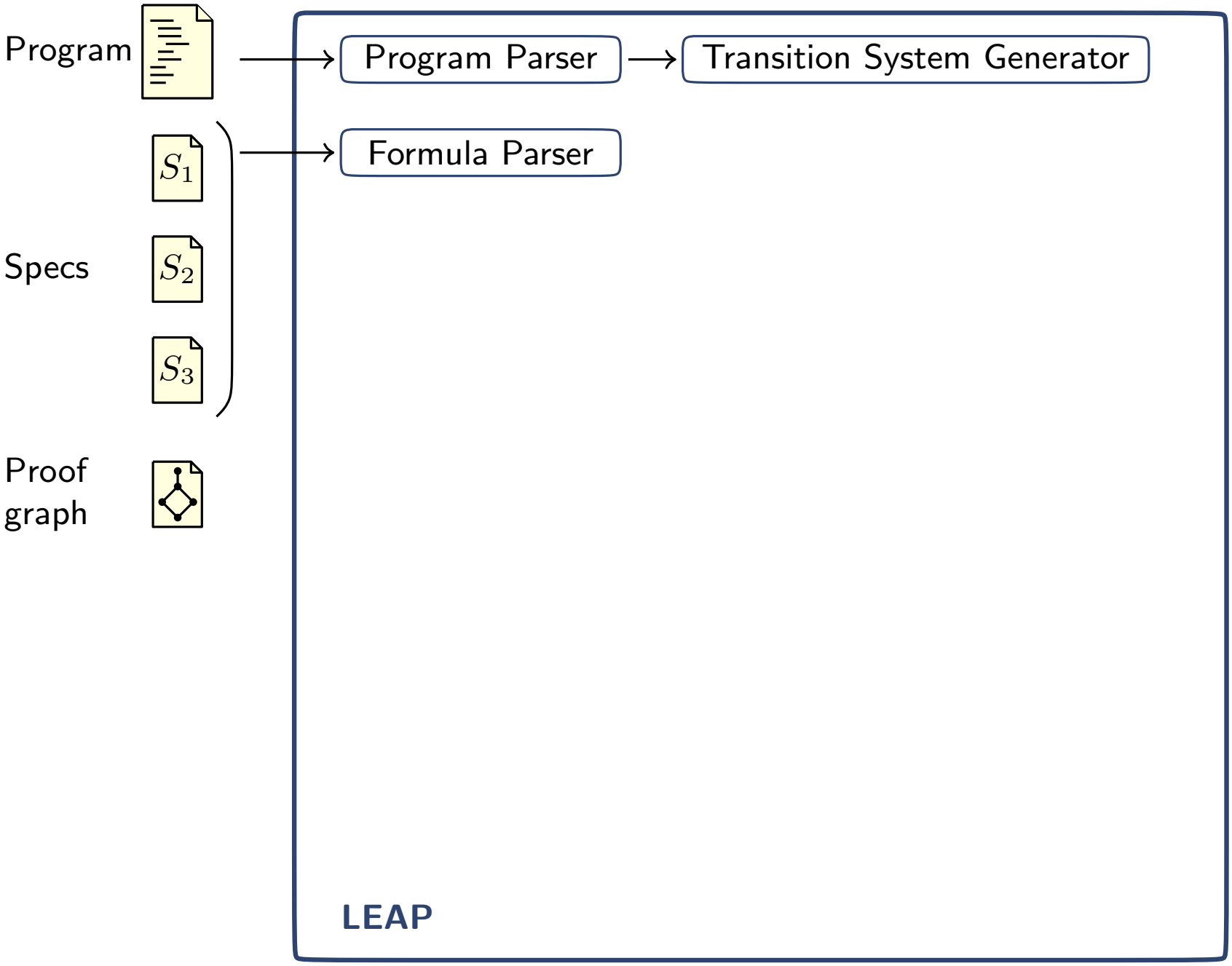
LEAP: Structure



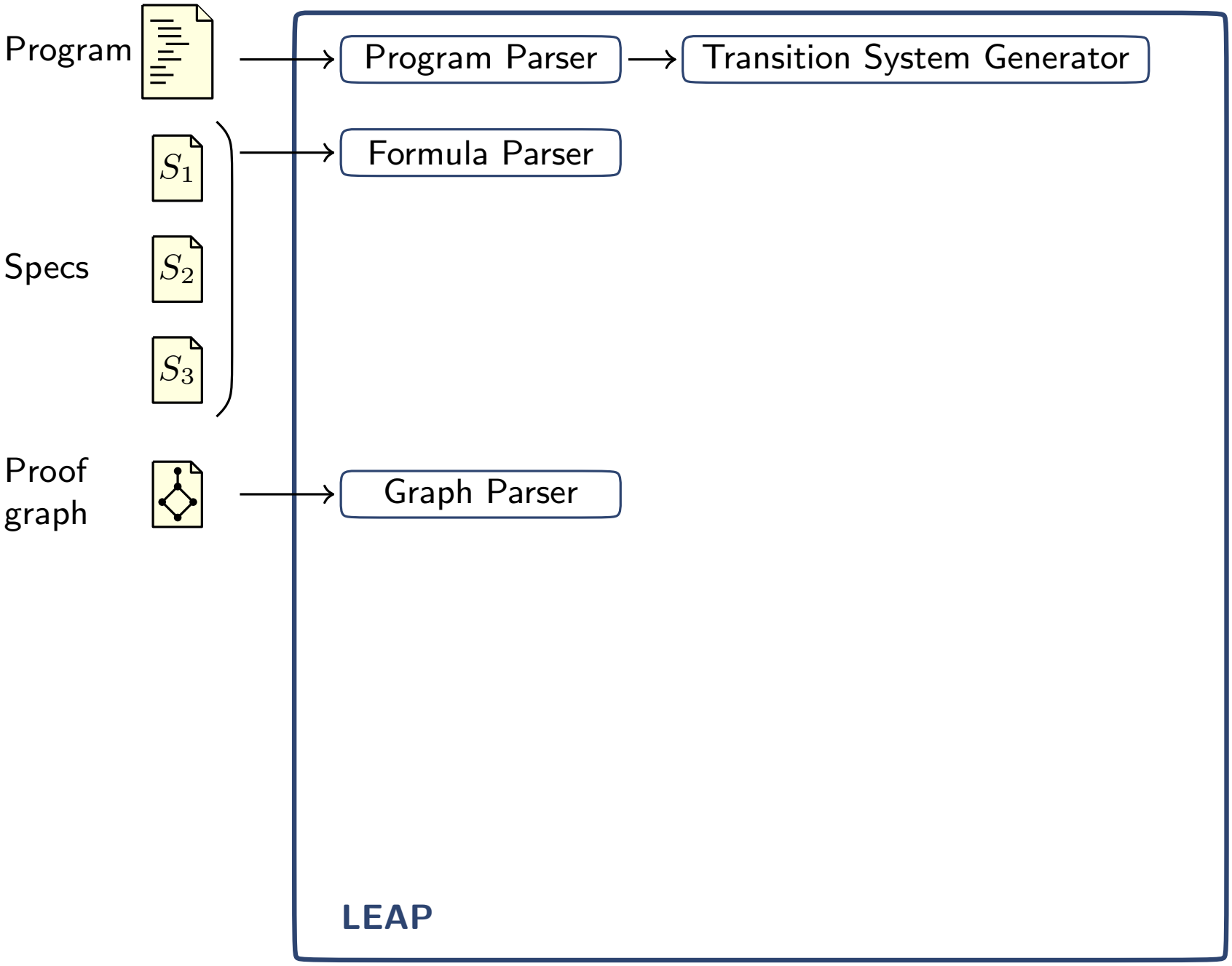
LEAP: Structure



LEAP: Structure



LEAP: Structure



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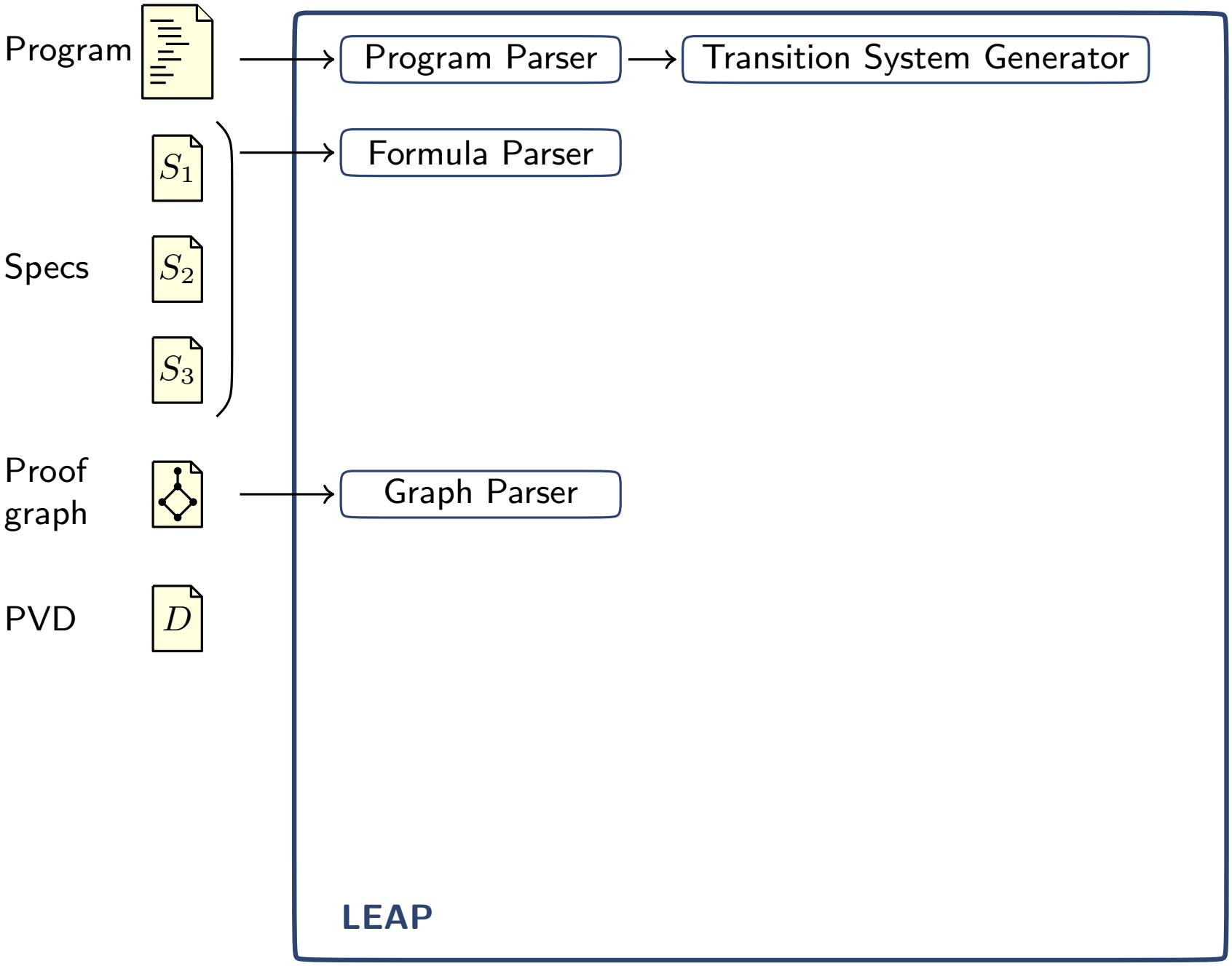
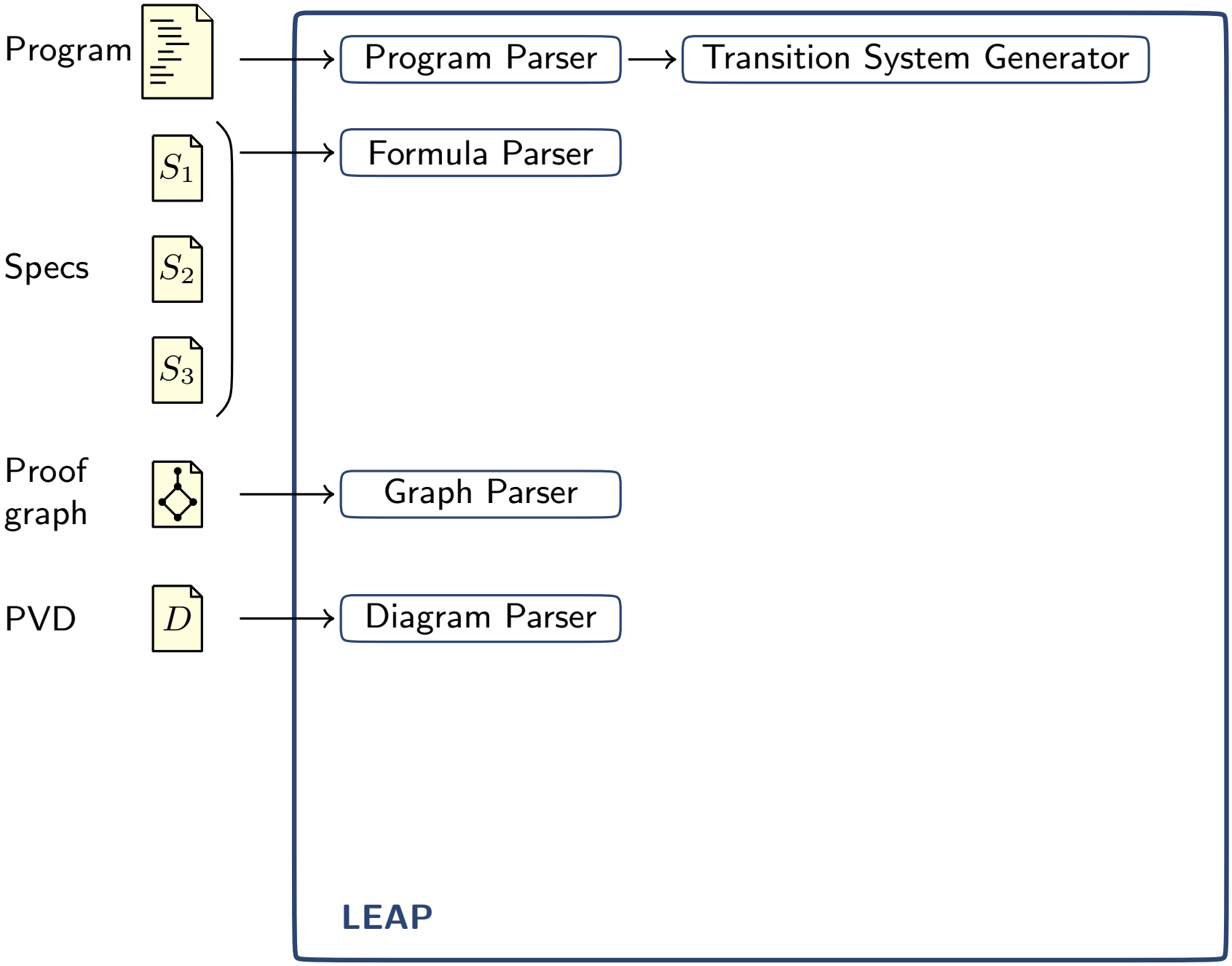


LEAP: Structure



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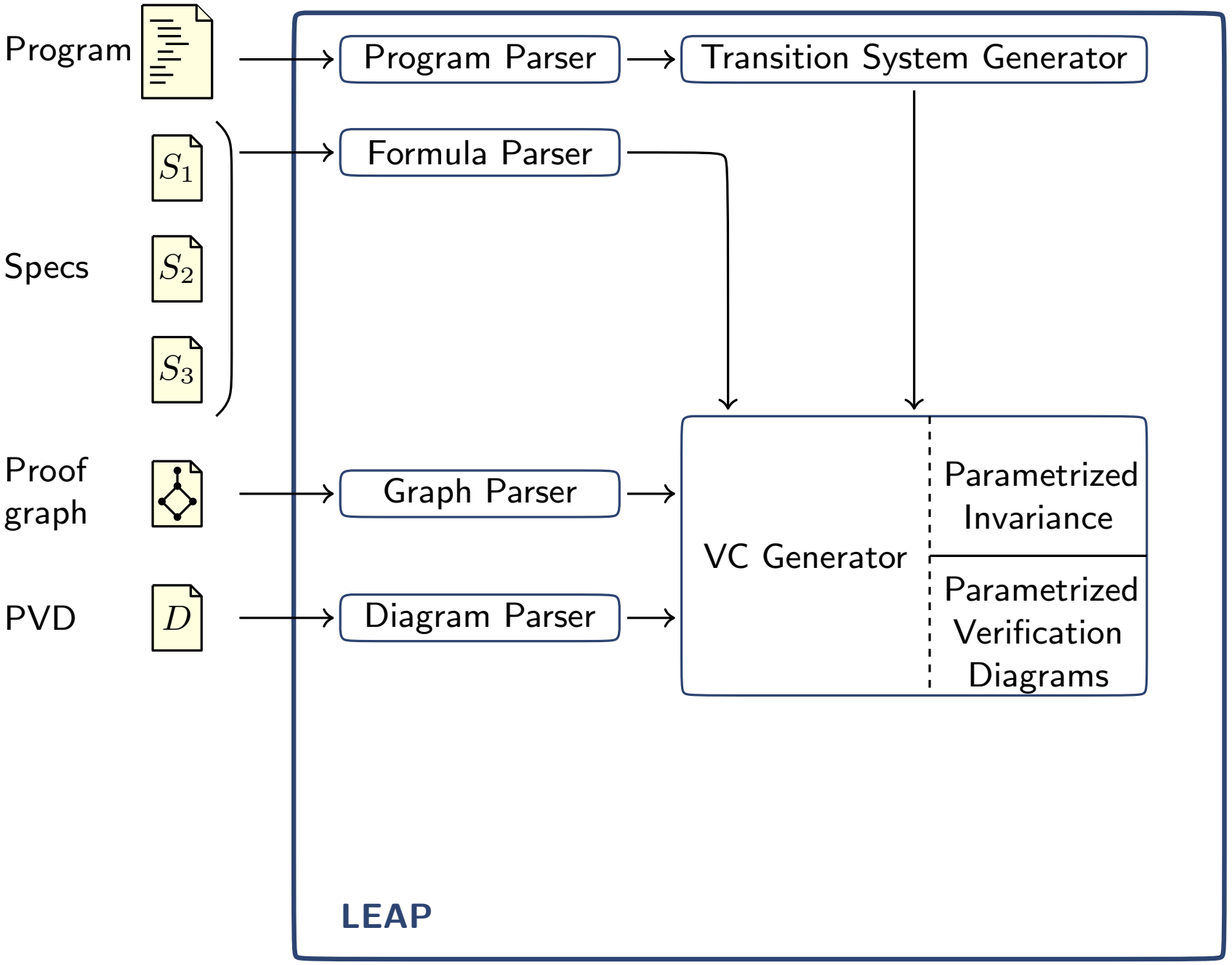
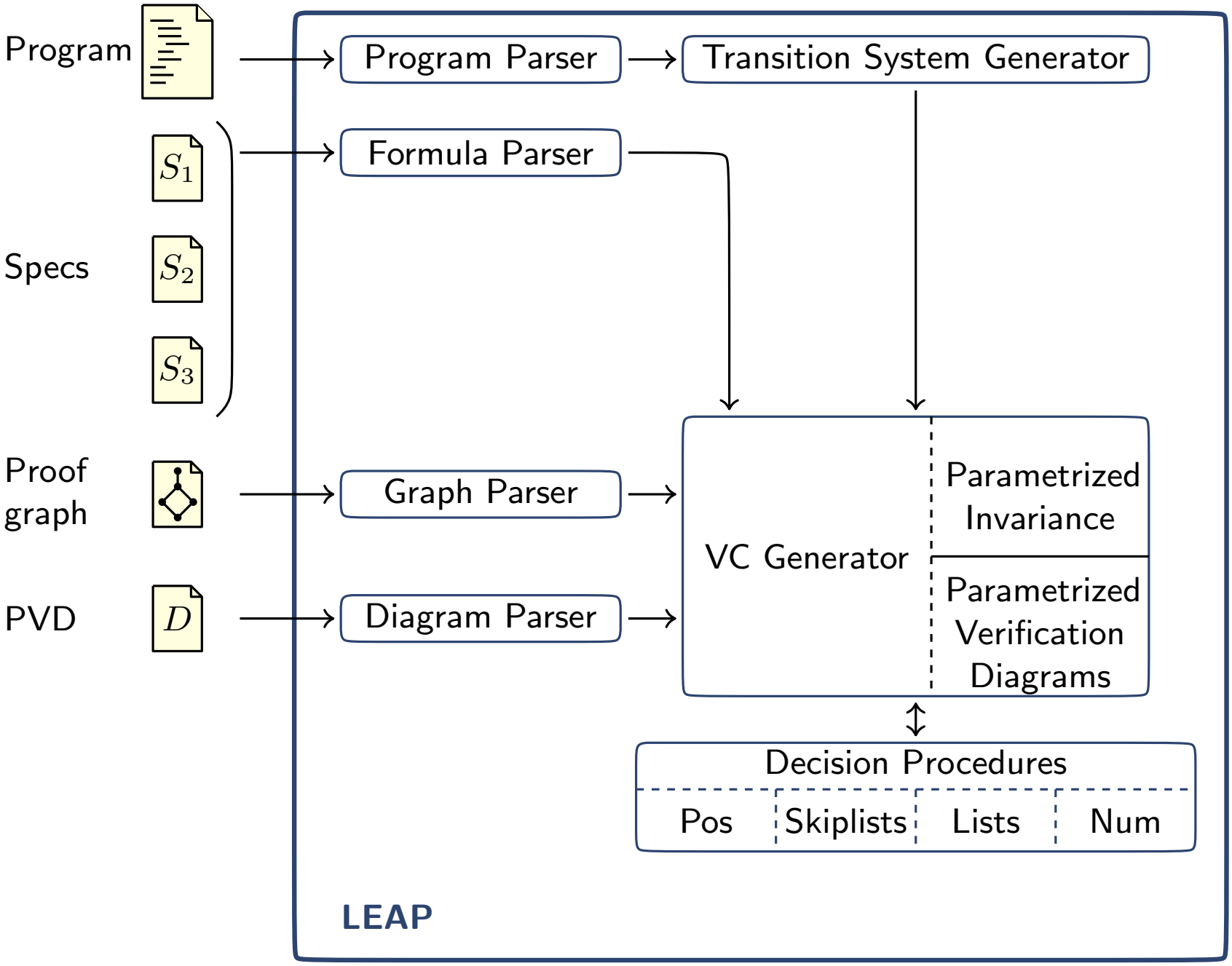
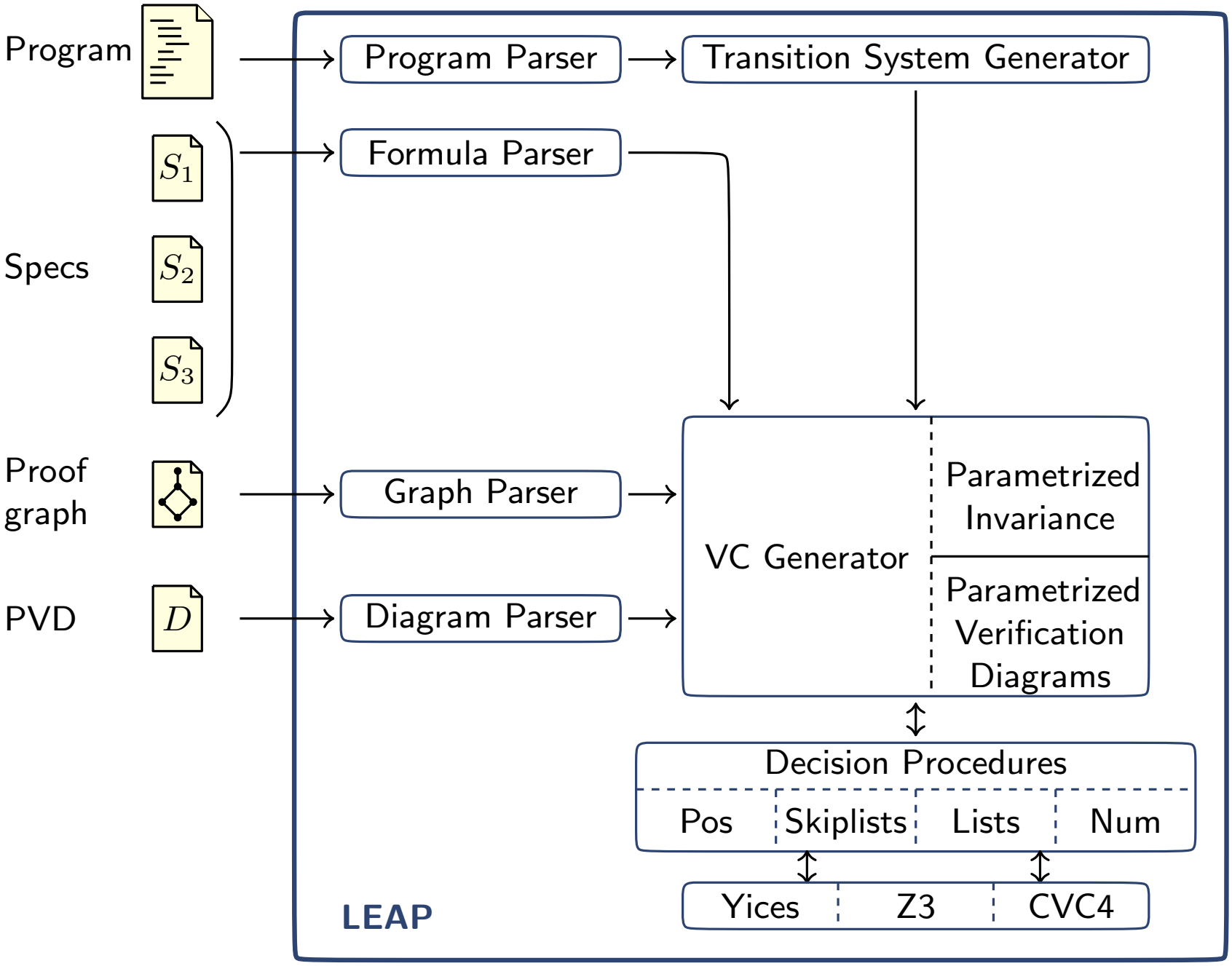


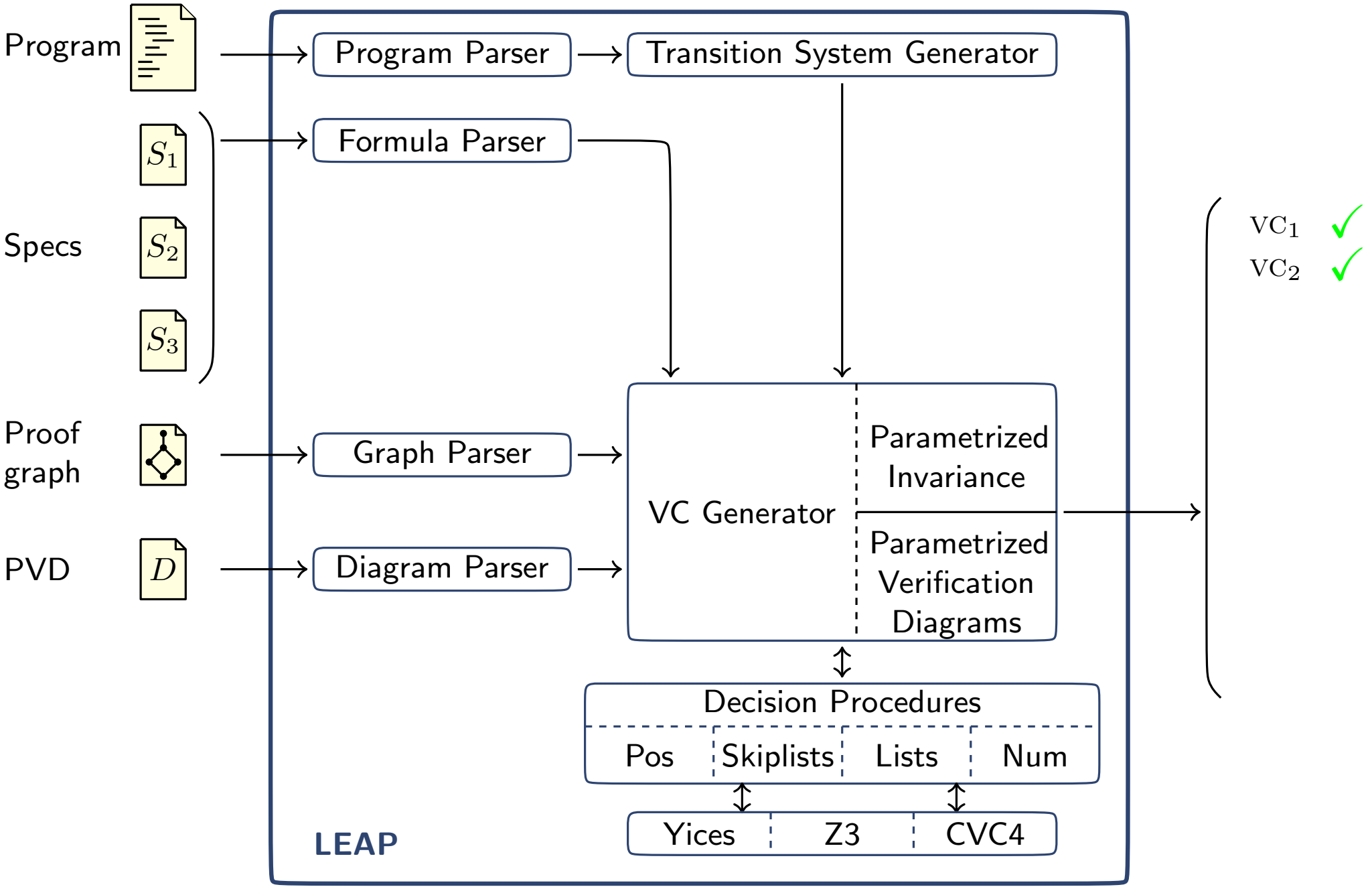
LEAP: Structure



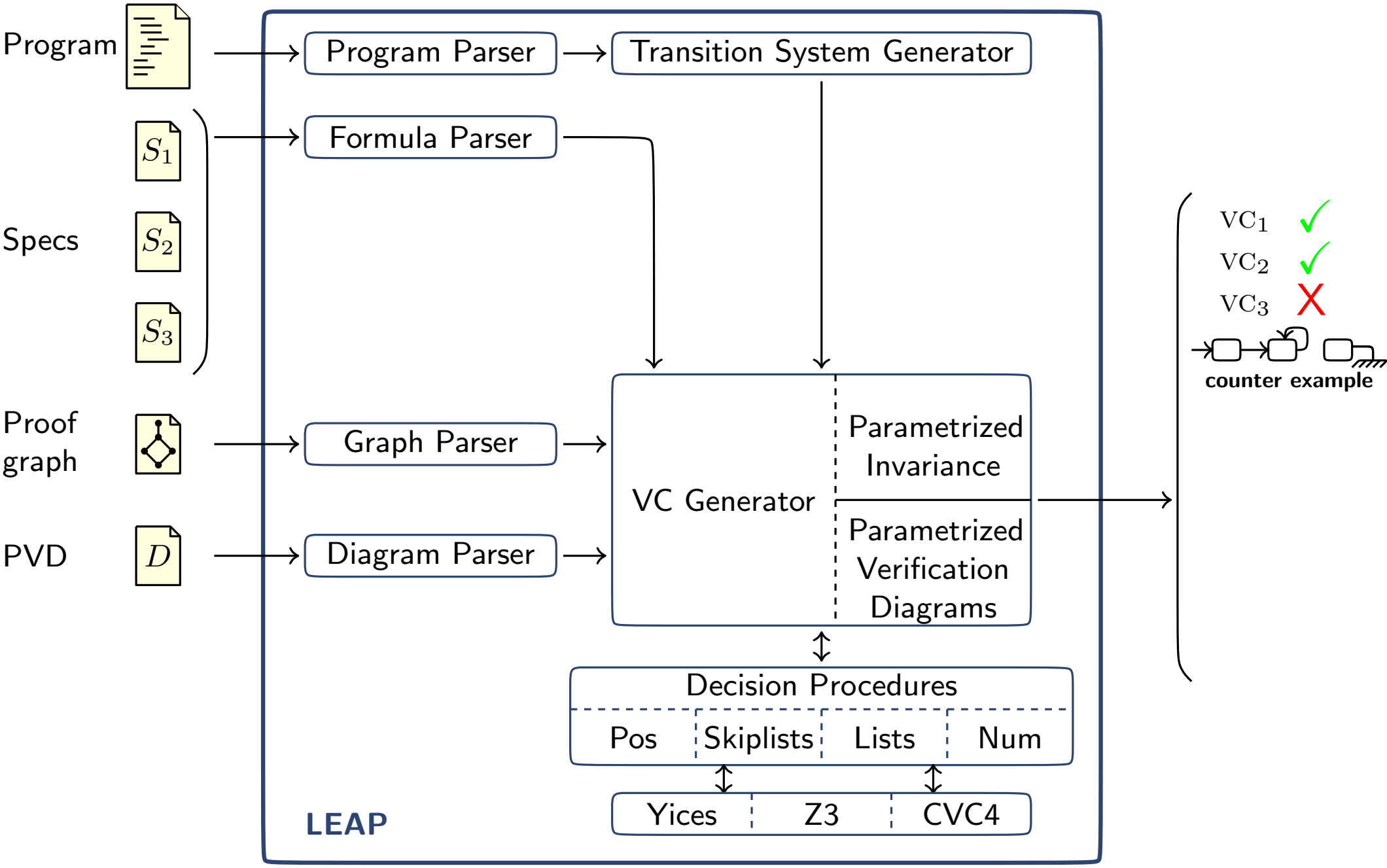
LEAP: Structure



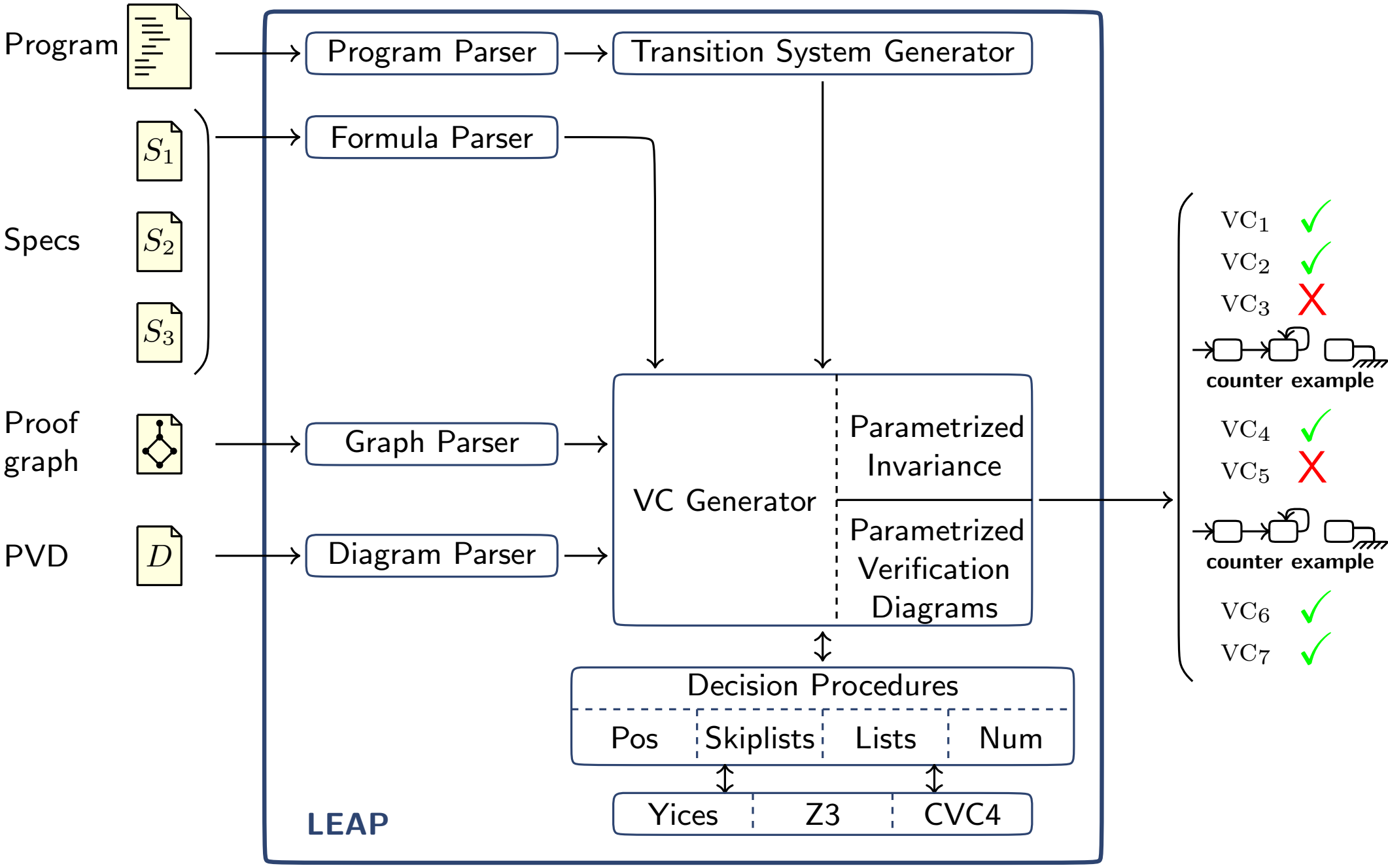
LEAP: Structure



LEAP: Structure



LEAP: Structure



Verification using LEAP

We use **LEAP** to verify temporal, structural and functional properties

- Mutual exclusion protocols
- Concurrent lock-coupling lists
- Concurrent lock-based queues
- Lock-free stacks
- Lock-free queues
- Skiplists with 2,3,4,5,.. levels
- Skiplists with unbounded levels (including KDE implementation)

LEAP and examples **available online** at

`software.imdea.org/leap`

LEAP: Some Experimental Results (safety)

		formula		#solved vc		Brute	Heurist.	DP time(s.)		LEAP
		idx	#vc	pos	dp	time(s.)	time(s.)	slowest	average	time(s.)
1	list	0	61	38	23	∞	18.67	11.90	0.30	0.20
2	order	1	121	62	59	998.35	1.12	0.03	0.01	0.47
3	lock	1	121	76	45	778.15	0.47	0.02	0.01	0.18
4	next	1	121	60	61	∞	2.11	0.61	0.01	0.59
5	region	1	121	95	26	∞	22.58	18.17	0.18	0.23
6	disj	2	181	177	4	121.74	0.19	0.01	0.01	0.12
7	funSchLin	1	121	97	24	∞	6.29	3.04	0.05	0.08
8	funSchIns	1	121	93	28	∞	4.15	1.91	0.03	0.08
9	funSchRem	1	121	93	28	∞	5.40	2.60	0.04	0.10
10	funSearch	1	208	198	10	∞	3.54	1.57	0.01	0.34
11	funInsert	1	208	200	8	∞	0.50	0.01	0.01	0.22
12	funRemove	1	208	200	8	∞	1.41	0.95	0.01	0.24
13	skiplist_3	0	154	92	62	∞	1221.97	776.45	15.27	0.45
14	region_3	0	124	97	27	∞	27.50	17.36	0.34	0.58
15	next_3	0	84	65	19	∞	0.67	0.09	0.01	0.20
16	order_3	0	84	59	25	∞	9.66	7.80	0.10	1.31
17	skiplist	0	560	532	28	∞	19.79	5.40	0.24	0.15
18	region	0	1583	1527	56	∞	44.28	22.66	0.54	1.35
19	next	0	1899	1869	30	∞	3.19	0.32	0.02	1.59
20	order	0	2531	2474	57	∞	11.19	2.35	0.84	6.75
21	mutex	2	28	26	2	0.32	0.01	0.01	0.01	0.01
22	minticket	1	19	18	1	0.04	0.01	0.01	0.01	0.01
23	notsame	2	28	26	2	0.13	0.03	0.01	0.01	0.01
24	mutexS	2	28	26	2	0.44	0.04	0.01	0.01	0.01
25	minticketS	1	19	18	1	0.31	0.01	0.01	0.01	0.01
26	notsameS	2	28	26	2	0.14	0.02	0.01	0.01	0.01

LEAP: Some Experimental Results (safety)

		formula		#solved vc		Brute	Heurist.	DP time(s.)		LEAP
		idx	#vc	pos	dp	time(s.)	time(s.)	slowest	average	time(s.)
1	list	0	61	38	23	∞	18.67	11.90	0.30	0.20
2	order	1	121	62	59	998.35	1.12	0.03	0.01	0.47
3	lock	1	121	76	45	778.15	0.47	0.02	0.01	0.18
4	next	1	121	60	61	∞	2.11	0.61	0.01	0.59
5	region	1	121	95	26	∞	22.58	18.17	0.18	0.23
6	disj	2	181	177	4	121.74	0.19	0.01	0.01	0.12
7	funSchLin	1	121	97	24	∞	6.29	3.04	0.05	0.08
8	funSchIns	1	121	93	28	∞	4.15	1.91	0.03	0.08
9	funSchRem	1	121	93	28	∞	5.40	2.60	0.04	0.10
10	funSearch	1	208	198	10	∞	3.54	1.57	0.01	0.34
11	funInsert	1	208	200	8	∞	0.50	0.01	0.01	0.22
12	funRemove	1	208	200	8	∞	1.41	0.95	0.01	0.24
13	skiplist_3	0	154	92	62	∞	1221.97	776.45	15.27	0.45
14	region_3	0	124	97	27	∞	27.50	17.36	0.34	0.58
15	next_3	0	84	65	19	∞	0.67	0.09	0.01	0.20
16	order_3	0	84	59	25	∞	9.66	7.80	0.10	1.31
17	skiplist	0	560	532	28	∞	19.79	5.40	0.24	0.15
18	region	0	1583	1527	56	∞	44.28	22.66	0.54	1.35
19	next	0	1899	1869	30	∞	3.19	0.32	0.02	1.59
20	order	0	2531	2474	57	∞	11.19	2.35	0.84	6.75
21	mutex	2	28	26	2	0.32	0.01	0.01	0.01	0.01
22	minticket	1	19	18	1	0.04	0.01	0.01	0.01	0.01
23	notsame	2	28	26	2	0.13	0.03	0.01	0.01	0.01
24	mutexS	2	28	26	2	0.44	0.04	0.01	0.01	0.01
25	minticketS	1	19	18	1	0.31	0.01	0.01	0.01	0.01
26	notsameS	2	28	26	2	0.14	0.02	0.01	0.01	0.01

LEAP: Some Experimental Results (safety)

		formula		#solved vc		Brute	Heurist.	DP time(s.)		LEAP
		idx	#vc	pos	dp	time(s.)	time(s.)	slowest	average	time(s.)
1	list	0	61	38	23	∞	18.67	11.90	0.30	0.20
2	order	1	121	62	59	998.35	1.12	0.03	0.01	0.47
3	lock	1	121	76	45	778.15	0.47	0.02	0.01	0.18
4	next	1	121	60	61	∞	2.11	0.61	0.01	0.59
5	region	1	121	95	26	∞	22.58	18.17	0.18	0.23
6	disj	2	181	177	4	121.74	0.19	0.01	0.01	0.12
7	funSchLin	1	121	97	24	∞	6.29	3.04	0.05	0.08
8	funSchIns	1	121	93	28	∞	4.15	1.91	0.03	0.08
9	funSchRem	1	121	93	28	∞	5.40	2.60	0.04	0.10
10	funSearch	1	208	198	10	∞	3.54	1.57	0.01	0.34
11	funInsert	1	208	200	8	∞	0.50	0.01	0.01	0.22
12	funRemove	1	208	200	8	∞	1.41	0.95	0.01	0.24
13	skiplist_3	0	154	92	62	∞	1221.97	776.45	15.27	0.45
14	region_3	0	124	97	27	∞	27.50	17.36	0.34	0.58
15	next_3	0	84	65	19	∞	0.67	0.09	0.01	0.20
16	order_3	0	84	59	25	∞	9.66	7.80	0.10	1.31
17	skiplist	0	560	532	28	∞	19.79	5.40	0.24	0.15
18	region	0	1583	1527	56	∞	44.28	22.66	0.54	1.35
19	next	0	1899	1869	30	∞	3.19	0.32	0.02	1.59
20	order	0	2531	2474	57	∞	11.19	2.35	0.84	6.75
21	mutex	2	28	26	2	0.32	0.01	0.01	0.01	0.01
22	minticket	1	19	18	1	0.04	0.01	0.01	0.01	0.01
23	notsame	2	28	26	2	0.13	0.03	0.01	0.01	0.01
24	mutexS	2	28	26	2	0.44	0.04	0.01	0.01	0.01
25	minticketS	1	19	18	1	0.31	0.01	0.01	0.01	0.01
26	notsameS	2	28	26	2	0.14	0.02	0.01	0.01	0.01

LEAP: Some Experimental Results (safety)

		formula		#solved vc		Brute	Heurist.	DP time(s.)		LEAP
		idx	#vc	pos	dp	time(s.)	time(s.)	slowest	average	time(s.)
1	list	0	61	38	23	∞	18.67	11.90	0.30	0.20
2	order	1	121	62	59	998.35	1.12	0.03	0.01	0.47
3	lock	1	121	76	45	778.15	0.47	0.02	0.01	0.18
4	next	1	121	60	61	∞	2.11	0.61	0.01	0.59
5	region	1	121	95	26	∞	22.58	18.17	0.18	0.23
6	disj	2	181	177	4	121.74	0.19	0.01	0.01	0.12
7	funSchLin	1	121	97	24	∞	6.29	3.04	0.05	0.08
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12	funRemove	1	208	200	8	∞	1.41	0.95	0.01	0.24
13	skiplist_3	0	154	92	62	∞	1221.97	776.45	15.27	0.45
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15	next_3	0	84	65	19	∞	0.67	0.09	0.01	0.20
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17	skiplist	0	560	532	28	∞	19.79	5.40	0.24	0.15
18	region	0	1583	1527	56	∞	44.28	22.66	0.54	1.35
19	next	0	1899	1869	30	∞	3.19	0.32	0.02	1.59
20	order	0	2531	2474	57	∞	11.19	2.35	0.84	6.75
21	mutex	2	28	26	2	0.32	0.01	0.01	0.01	0.01
22	minticket	1	19	18	1	0.04	0.01	0.01	0.01	0.01
23	notsame	2	28	26	2	0.13	0.03	0.01	0.01	0.01
24	mutexS	2	28	26	2	0.44	0.04	0.01	0.01	0.01
25	minticketS	1	19	18	1	0.31	0.01	0.01	0.01	0.01
26	notsameS	2	28	26	2	0.14	0.02	0.01	0.01	0.01

LEAP: Some Experimental Results (safety)

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3	lock	1	121	76	45	778.15	0.47	0.02	0.01	0.18
4	next	1	121	60	61	∞	2.11	0.61	0.01	0.59
5	region	1	121	95	26	∞	22.58	18.17	0.18	0.23
6	disj	2	181	177	4	121.74	0.19	0.01	0.01	0.12
7	funSchLin	1	121	97	24	∞	6.29	3.04	0.05	0.08
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10	funSearch	1	208	198	10	∞	3.54	1.57	0.01	0.34
11	funInsert	1	208	200	8	∞	0.50	0.01	0.01	0.22
12	funRemove	1	208	200	8	∞	1.41	0.95	0.01	0.24
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15	next_3	0	84	65	19	∞	0.67	0.09	0.01	0.20
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18	region	0	1583	1527	56	∞	44.28	22.66	0.54	1.35
19	next	0	1899	1869	30	∞	3.19	0.32	0.02	1.59
20	order	0	2531	2474	57	∞	11.19	2.35	0.84	6.75
21	mutex	2	28	26	2	0.32	0.01	0.01	0.01	0.01
22	minticket	1	19	18	1	0.04	0.01	0.01	0.01	0.01
23	notsame	2	28	26	2	0.13	0.03	0.01	0.01	0.01
24	mutexS	2	28	26	2	0.44	0.04	0.01	0.01	0.01
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LEAP: Some Experimental Results (safety)

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3	lock	1	121	76	45	778.15	0.47	0.02	0.01	0.18
4	next	1	121	60	61	∞	2.11	0.61	0.01	0.59
5	region	1	121	95	26	∞	22.58	18.17	0.18	0.23
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7	funSchLin	1	121	97	24	∞	6.29	3.04	0.05	0.08
8	funSchIns	1	121	93	28	∞	4.15	1.91	0.03	0.08
9	funSchRem	1	121	93	28	∞	5.40	2.60	0.04	0.10
10	funSearch	1	208	198	10	∞	3.54	1.57	0.01	0.34
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13	skiplist_3	0	154	92	62	∞	1221.97	776.45	15.27	0.45
14	region_3	0	124	97	27	∞	27.50	17.36	0.34	0.58
15	next_3	0	84	65	19	∞	0.67	0.09	0.01	0.20
16	order_3	0	84	59	25	∞	9.66	7.80	0.10	1.31
17	skiplist	0	560	532	28	∞	19.79	5.40	0.24	0.15
18	region	0	1583	1527	56	∞	44.28	22.66	0.54	1.35
19	next	0	1899	1869	30	∞	3.19	0.32	0.02	1.59
20	order	0	2531	2474	57	∞	11			6.75
21	mutex	2	28	26	2	0.32	0		1	0.01
22	minticket	1	19	18	1	0.04	0		1	0.01
23	notsame	2	28	26	2	0.13	0.03	0.01	0.01	0.01
24	mutexS	2	28	26	2	0.44	0.04	0.01	0.01	0.01
25	minticketS	1	19	18	1	0.31	0.01	0.01	0.01	0.01
26	notsameS	2	28	26	2	0.14	0.02	0.01	0.01	0.01

LEAP analysis time remains insignificant

LEAP: Some Experimental Results (safety)

		formula		#solved vc		Brute	Heurist.	DP time(s.)		LEAP
		idx	#vc	pos	dp	time(s.)	time(s.)	slowest	average	time(s.)
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2	order	1	121	62	59	998.35	1.12	0.03	0.01	0.47
3	lock	1	121	76	45	778.15	0.47	0.02	0.01	0.18
4	next	1	121	60	61	∞	2.11	0.61	0.01	0.59
5	region	1						18.17	0.18	0.23
6	disj	2						0.01	0.01	0.12
7	funSchLin	1						3.04	0.05	0.08
8	funSchIns	1	121	93	28	∞	4.15	1.91	0.03	0.08
9	funSchRem	1	121	93	28	∞	5.40	2.60	0.04	0.10
10	funSearch	1	208	198	10	∞	3.54	1.57	0.01	0.34
11	funInsert	1	208	200	8	∞	0.50	0.01	0.01	0.22
12	funRemove	1	208	200	8	∞	1.41	0.95	0.01	0.24
13	skiplist_3	0	154	92	62	∞	1221.97	776.45	15.27	0.45
14	region_3	0	124	97	27	∞	27.50	17.36	0.34	0.58
15	next_3	0	84	65	19	∞	0.67	0.09	0.01	0.20
16	order_3	0	84	59	25	∞	9.66	7.80	0.10	1.31
17	skiplist	0	560	532	28	∞	19.79	5.40	0.24	0.15
18	region	0	1583	1527	56	∞	44.28	22.66	0.54	1.35
19	next	0	1899	1869	30	∞	3.19	0.32	0.02	1.59
20	order	0	2531	2474	57	∞	11.19	2.35	0.84	6.75
21	mutex	2	28	26	2	0.32	0.01	0.01	0.01	0.01
22	minticket	1	19	18	1	0.04	0.01	0.01	0.01	0.01
23	notsame	2	28	26	2	0.13	0.03	0.01	0.01	0.01
24	mutexS	2	28	26	2	0.44	0.04	0.01	0.01	0.01
25	minticketS	1	19	18	1	0.31	0.01	0.01	0.01	0.01
26	notsameS	2	28	26	2	0.14	0.02	0.01	0.01	0.01

Decision procedures perform well... but still room for improvements

LEAP: Some Experimental Results (liveness)

MinTicket (progress):

	#VC	#solved VC		single VC time(s.)		DP	LEAP
		pos	num	slowest	average	time(s)	time(s)
Initiation	1	0	1	0.01	0.01	0.01	0.01
Consecution	153	144	9	2.66	0.03	4.22	0.06
Acceptance	195	132	63	1.46	0.08	15.28	0.05
Fairness	24	20	4	0.03	0.01	0.10	0.02

Concurrent List (termination):

	#VC	#solved VC		single VC time(s.)		DP	LEAP
		pos	TLL	slowest	average	time(s)	time(s)
Initiation	1	0	1	0.01	0.01	0.01	0.01
Consecution	1550	1343	207	3.80	0.05	78.12	3.42
Acceptance	5404	4352	1052	191.61	0.12	647.04	1.61
Fairness	48	20	28	0.42	0.16	7.82	0.14

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Theory and decision procedure for skiplists with unbounded many levels

C Implementation and Evaluation of our Framework

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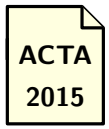

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Conclusions

- A novel **deductive framework** for parametrized verification
- Designed for **concurrent data structures**
- Suitable for **safety** and **liveness** temporal properties
- Constructed **specialized theories and decision procedures**
- We studied **automatic generation** of parametrized invariants
- **Implemented a tool** with our verification framework
- We **verified various programs** and concurrent data structures

Future Work and Open Questions

- **Relax symmetry** and study **process topologies**
- Better **invariant generation**: for more data domains
- Beyond interleaving semantics:
  Weak memory models and **distributed algorithms**
- More **decision procedures** for more datatypes
- Implement a generic **Nelson-Oppen** for faster decision procedures
- Use **theorem provers** in combination with SMT solvers, and generate full formal proofs
- Add support for **real world programming languages**

Future Work

- **Relax symmetry** and study **process topologies**

To show that $\mathcal{S}$ satisfies $\varphi(i)$:

$$\frac{\begin{array}{lrcll} (\ \mathsf{I}\ ) & \Theta & \rightarrow & \varphi & \\ (\mathsf{SC}) & \varphi \wedge \tau^{(i)} & \rightarrow & \varphi' & \text{forall } \tau \\ (\mathsf{OC}) & \varphi \wedge k \neq i \wedge \tau^{(k)} & \rightarrow & \varphi' & \text{forall } \tau, \text{fresh } k \end{array}}{\Box\varphi}$$

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$$\begin{array}{lrcll} (\mathsf{OC}_<) & \varphi \wedge k < i \wedge \tau^{(k)} & \rightarrow & \varphi' & \text{forall } \tau, \text{fresh } k \\ (\mathsf{OC}_>) & \varphi \wedge k > i \wedge \tau^{(k)} & \rightarrow & \varphi' & \text{forall } \tau, \text{fresh } k \end{array}$$

Thanks!